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Free vibration analysis of two-dimensional functionally graded structures by a meshfree boundary–domain integral equation method

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ABSTRACT

Free vibration of two-dimensional functionally graded structures with an exponential material gradation is analyzed in this paper by a meshfree boundary–domain integral equation method. Based on the two-dimensional elasticity theory, boundary–domain integral equations are derived by using elastostatic fundamental solutions. Due to the material inhomogeneity and inertial effect, two domain integrals emerge in the boundary–domain integral equation formulation. Radial integration method is employed to convert the domain integrals into boundary integrals. A meshfree scheme is achieved through approximating the normalized displacements in the domain integrals by a combination of the radial basis functions and the polynomials. Thus, the free vibration problem is reduced into a generalized eigenvalue problem, which involves system matrices with boundary integrals only. By using the developed meshfree boundary–domain integral equation method, free vibration of two-dimensional exponentially graded beams and plates with various material gradients, gradation directions, boundary conditions and aspect ratios is investigated, which demonstrates the high convergence, efficiency and accuracy of the present method.

1. Introduction

Functionally graded materials (FGMs) belong to an advanced class of composite materials. In order to optimize the material properties, FGMs could be produced from completely different constituent combinations that fit the required structural functions with a high structural efficiency. Continuous gradation of material properties reduce the residual stresses and stress concentrations which are fatal defects existing in traditional composites. Besides, FGMs bring extraordinary merits of high resistance to temperature gradients, high wear resistance and an increase in strength to weight ratio, which make the FGMs have extensive applications. Example applications include the increased relevance of the FGM structural components in the design of industry constructions, aerospace structures, optoelectronics components and fusion energy devices. Comprehensive reviews of FGMs research can be found in the books by Suresh and Mortensen [1], Miyamoto [2] and Robert et al. [3].

The superior properties of FGMs attract many research interests of material scientists and structural designers. Studies on modelling the dynamic behaviours of FGM become more important in engineering design procedures and free vibration analysis is the first step on the way to explore vibration problems. A number of researches about free vibration of FGMs structures like beams and plates have been reported in the literature. Qian and Ching [4] analyzed the static deformation, free and forced vibration of a two-dimensional (2D) functionally graded cantilever beam by using meshless local Petrov–Galerkin (MLPG) method. The material properties of the beam change with a power-law variation proportional to the volume fraction of the constituents in the thickness and longitudinal directions. Şimşek and Kocatürk [5] analyzed free vibration characteristics and the dynamic behaviour of a simply-supported FG beam under a concentrated moving harmonic load. The system of equations of motion is derived by using Lagrange's equations under the assumptions of the Euler–Bernoulli beam theory. Both exponential and power laws are applied for the material properties varying continuously in the thickness direction. Alshorbagy et al. [6] presented the dynamic characteristics of a functionally graded beam with a material graduation in axial or transversal (thickness) directions with power law function by finite element method.

Zhao et al. [7] used the element-free *kp*-Ritz method and the first order shear deformation plate theory to analyze the free vibration of functionally graded plates. The material properties are assumed to vary continuously through the thickness according to a power law distribution of the volume fractions of the plate constituents. Ferreira et al. [8] employed the global collocation method, the first and the third-order shear deformation plate

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theories to analyze the free vibration of functionally graded plates. The material properties were assumed to vary in the thickness direction computed by the Mori–Tanaka homogenization technique. Liu et al. [9] investigated the free vibration of FGM plates with an in-plane material inhomogeneity based on the classical plate theory. A plate material gradation with a power-law along one in-plane direction is considered. It is shown when the simplified structural theories are employed (i.e. plate theories), there is no significant difference in the analysis between FGM plates and the conventional laminated plates.

Analytical methods have been also applied for vibration analysis of FGMs and structures. Aydogdu and Taskin [10] obtained the analytical results of free vibration of simply supported FG beams by using Hamilton's principle in which Young's modulus of the beam varies in the thickness direction according to power and exponential laws. Different higher order shear deformation theories and classical beam theories are used in the analysis. An analytical method for free vibration analysis of functionally graded beam was investigated by Sina et al. [11]. The equations of motion of FG beams are derived using a new beam theory and Hamilton's principle. The variation of the FG beam properties is assumed to have a simple power law function of the volume fractions of the material constituents of the beam. Li et al. [12] presented an analytic treatment of the free vibration of axially exponentially graded beams with various support conditions. The analytical solutions for FG plates are quite rare in the literature. Vel and Batra [13] obtained an exact solution for the vibration of simply supported rectangular thick plates with material properties varying in the thickness direction only. Based on the classical plate theory, an exact analytical solution for free vibration of thin FG rectangular plates was presented by Baferani et al. [14].

Analytical solutions can be served as a benchmark for assessing the accuracy and efficiency of various numerical and approximate approaches. However, due to the mathematical complexity, most practical problems of FGM structures can be solved only with numerical schemes. Among many numerical methods, boundary element method (BEM) or boundary integral equation method (BIEM) with its superior efficiency and accuracy has been well established as a powerful method for static and dynamic analysis of FGMs and structures. Masataka et al. [15] solved the steady-state heat conduction problems of FGMs by a dual reciprocity boundary element method (DRBEM). Fracture analysis of FGMs by a BEM was presented in [16,17]. Ruocco and Minutolo [18] investigated two-dimensional stress analysis of multi-region functionally graded materials using a field boundary element model. Damanpack et al. [19] applied the BEM to the bending analysis of functionally graded plates.

In the dynamic analyses of FGMs and structures by BEM, three approaches are often used, namely, time domain method (TDM), Laplace-transform method (LTM), and dual reciprocity method (DRM) [20]. The fundamental solutions required in the TDM and LTM have a complicated form, and the computational efficiency of the method is therefore significantly reduced. In the DRM, the equations of motion are expressed in a boundary integral form using the fundamental solutions of elastostatics, which involves domain integrals to take the inertial effects into account. The domain integrals are transformed into boundary integrals by the DRM [21]. Since the fundamental solutions of elastostatics are time independent, therefore only space integration is required. The DRM requires particular solutions, whose construction is restricted to the approximation function chosen. Another efficient alternative technique to handle the domain integrals in BEM is the radial integration method (RIM) [22]. RIM is based on pure mathematical treatments, and the main advantage over the DRM is that the radial basis functions (RBF) can be freely chosen. The DRM and RIM have been successfully implemented and applied to elastodynamic problems in homogeneous and anisotropic materials [23,24]. A meshless boundary–domain integral equation method for transient thermoelastic analysis has been developed and applied by Ekhlakov et al. [25,26], where the Laplace-transform technique is used and the domain integrals are handled by the RIM. In contrast to the above mentioned investigations, few works can be found in literature for the elastodynamic problems of FGM structures by using these methods.

In this paper a meshfree boundary–domain integral equation method is presented to analyze the free vibration behaviours of 2D FGM structures. The material properties are assumed to vary continuously according to the exponential law either in thickness or longitudinal directions. The boundary–domain integral equations are derived based on 2D elasticity theory, while the elastostatic fundamental solutions for isotropic and homogeneous materials are applied. Radial integration method is applied to convert the two domain integrals emerged in the boundary–domain integral equations due to the material inhomogeneity and inertial effect into the boundary integrals. A meshfree scheme is achieved by approximating the normalized displacements in the domain integrals by a combination of the radial basis function and polynomials in term of global coordinates. Internal nodes are necessary to increase the accuracy of the solution due to the fact that the correct functional approximation requires a rather uniform distribution of nodal points. Then the eigenvalue and eigenvector can be obtained by solving the generalized eigensystem with only boundary discretization and internal nodes. The present method keeps the dimensionality reduction advantage of the classical BEM and use simple elastostatic fundamental solutions. Numerical examples demonstrate its high efficiency and accuracy.

2. Exponential material properties

In this paper, the material properties of the FGM structures are assumed to vary continuously along spatial coordinates in longitudinal or thickness directions according to the exponential law. In practice, the Poisson's ratio ν is usually taken as constant since it commonly varies only slightly within the material and this assumption was used often [27]. The Young's modulus E and the mass density ρ are assumed to vary according to the following exponential functions [28]

$$E(x_d) = E_0 e^{\beta x_d}, \quad \text{where } \beta = \frac{1}{L_d} \ln\left(\frac{E_1}{E_0}\right), \tag{1}$$

$$\rho(x_d) = \rho_0 e^{\gamma x_d}, \quad \text{where } \gamma = \frac{1}{L_d} \ln\left(\frac{\rho_1}{\rho_0}\right), \tag{2}$$

where E_0 and ρ_0 denote the Young's modulus and mass density for the starting face constituent, E_1 and ρ_1 are for the ending face constituent, β and γ represent the material gradient parameters for Young's modulus and mass density respectively, x_d stands for the Cartesian coordinate system, and L_d is length parameter ($d = 1, 2$) of the considered structure. An example for the variation of the exponential material properties in spatial direction as $E(x_d)$ is shown in Fig. 1, where $\kappa = E_1/E_0$.

3. Problem formulation

Based on 2D elasticity theory, the governing equations of motion are given by

$$\sigma_{ij,j}(\boldsymbol{x}, t) = \rho \ddot{u}_i(\boldsymbol{x}, t), \tag{3}$$

where ρ is the mass density, u_i is the displacement vector, $\ddot{u}_i = \partial^2 u_i(\boldsymbol{x}, t)/\partial t^2$ is the acceleration, and σ_{ij} is the stress tensor. A comma after a quantity represents spatial derivatives and repeated