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Nonlinear resonance behavior of functionally graded cylindrical shells in thermal environments

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ABSTRACT

This paper deals with the nonlinear vibrations of functionally graded cylindrical shells in thermal environments. The equivalent properties of functionally graded materials are described as a power-law distribution in the thickness direction and are considered to be temperature-dependent. A typical case with a primary resonance excitation and a 1:2 internal resonance between two modes is analyzed. The energy approach and the Lagrangian formulation are employed to derive the reduced low-dimensional nonlinear ordinary differential equations of motion based on Donnell's nonlinear shell theory. The dynamic behaviors of system are investigated by means of the so-called multiple scale method. The amplitude–frequency curves and the bifurcation behavior of the system are analyzed using numerical continuation method. The effects of temperature and volume fractions of constituent material on the amplitude response of the system are fully discussed.

1. Introduction

By gradual changing the volume fraction of constituent materials through special directions, the material properties of functionally graded materials (FGM) are smoothly variational along one or more directions and can response to externally applied loading in optimum way [1]. Such advantages induce FGMs attract intensive attention in a wide range of research subjects. During the past one and a half decades, numerous reports on the studies of vibrations or dynamic response of functionally grade (FG) structures have been published (e.g., Refs. [2–9]). To characterize the gradient effects of FGMs as precise as possible, many researchers developed higher order shear deformation theory to modify the classic plate or shell theory. Yang and Shen [4], Huang and Shen [5] studied the free vibration and dynamic response of FG cylindrical panels and plates based on a higher order shear deformation shell and plate theory respectively. Patel et al. [7] analyzed the free vibration of FG elliptical cylindrical shells with higher order theory. The others who consider the shear deformation in the study of vibration of FG structures include Chen [6], Matsunaga [8,10,11], Talha and Singh [9], Hosseini-Hashemi et al. [12], Zhao and Liew [13]. On the other hand, some researchers try to investigate the dynamic problems of FG structures directly based on three dimensional (3D) theory of elasticity. Malekzadeh [14] analyzed 3D free vibration of FG plates resting on an elastic medium, Malekzadeh and co-workers [15,16] also studied 3D free vibration of FG thick annular plates and truncated conical shells respectively. Li et al. [17] gave a solution for the free vibration of FG rectangular plates with simply supported and clamped edges according to 3D linear theory of elasticity. Vel [18] presented an exact elastic solution for the vibration of FG anisotropic cylindrical shells based on the 3D linear elastodynamics. Asgari and Akhlaghi [19] presented a natural frequency analysis of thick hollow cylinders made of two dimensional FGM according to 3D equations of elasticity. Meanwhile, many solution methods are developed to treat FG structures. Pradyumna and Bandyopadhyay [20] proposed a higher-order finite element formulation to analyze the free vibration of FG curved panels; Talha and Singh [9], Natarajan et al. [21], Behjat and Khoshravan [22] also applied finite element method (FEM) to study the free vibration of FG plates. Analytical approach [5,23,24] and differential quadrature method (DQM) [12,25–28] are also used in vibration analysis of FG plates or shells.

Since FGMs are developed primarily for use in high temperature environments, thermal effects on the dynamic response of FG structures are always significant aspects. Huang and Shen [5] revealed the temperature field has significant effect on the nonlinear vibration and dynamic response of FG plates by taking both heat conduction and temperature-dependent material properties into account. Yang and Shen [25] studied free and forced vibration of FG plates in thermal environments. Malekzadeh and co-workers [15,16] investigated 3D free vibration of FG annular plates and truncated conical shells in thermal environments; they also presented a solution for temperature-dependent free vibration of FG rotating cylindrical shells [29].

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Recently, some researchers treated the nonlinear dynamic behaviors of FG plates and shells, such as bifurcation, mode interaction, and chaotic motion. Alijani et al. [30] analyzed the nonlinear forced vibration of FG doubly-curved shallow shells, the primary and subharmonic resonance responses of FG shallow shells were fully discussed; bifurcation diagrams and the Poincaré maps were obtained, and chaotic regions were illustrated. They further discussed the thermal effects of temperature on the nonlinear vibrations of FG doubly curved shells [31]. Alijani and co-workers [32] also investigated the nonlinear vibrations of FG plates in thermal environment, and revealed significant effect of temperature on the vibration of FG plates. Hao et al. [33] analyzed nonlinear dynamic behaviors of cantilever FG rectangular plates in thermal environment, and numerical results showed that cantilever FG rectangular plates may occur periodic, quasi-periodic and chaotic motion in some given conditions.

This work deals with the nonlinear forced vibration of infinitely long FG cylindrical thin shells with thermal effects. The properties of FGM are assumed to be temperature-dependent and graded in the thickness direction according to a simple power-law distribution in terms of the volume fractions of constituents. The Donnell's nonlinear shell theory and the Lagrangian formulation are employed to derive the reduced low-dimensional nonlinear ordinary differential equations of motion of system. The multiple scale method is applied to analyze the nonlinear resonance behaviors of the shell. The effects of temperature and power-law exponent on the amplitude response of the system are fully investigated.

2. Basic equations

2.1. Equations of motion

Consider a FG cylindrical thin shell with mid-surface radius R and thickness h. A reference frame of cylindrical coordinates (x,θ,z) is set up at the middle surface, where x is longitudinal, θ circumferential, z normal (positive outwards). The deformations of mid-surface defined in the reference frame are u, v, and w in the x, θ, and z directions, respectively.

Assume that the temperature is uniform across the thickness and can be expressed as $T = T_0 + \Delta T$, where $T_0 = 300$ K is the initial temperature, and ΔT the temperature rise. The constituent materials of FG cylindrical shells are assumed to be metal and ceramic; and their properties P (Young's modulus E, Poisson's ratio v, mass density ρ and thermal expansion coefficient α_T) are considered as temperature-dependent [2]:

$$P(T) = P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right) \tag{1}$$

The material properties P of FGM are supposed to be graded in thickness direction and are expressed as functions of z-coordinate in terms of a power-law distribution [2]:

$$P(z,T) = [P_m(T) - P_c(T)]\left(\frac{2z+h}{2h}\right)^N + P_c(T) \tag{2}$$

where the subscripts m and c represent metal and ceramic, respectively; the superscript N is power-law exponent, $N \in [0,\infty)$, reflecting the volume fraction of metal. According to Eq. (2), the material of inner surface ($z = -h/2$) is ceramic, and the outer surface ($z = h/2$) is metal. The temperature dependent material properties of metal and ceramic are shown in Table 1.

Here, we introduce some material moduli which will be used in following analysis:

$$\bar{\rho} = \int_{-h/2}^{h/2} \rho(z,T)\mathrm{d}z \tag{3a}$$

$$\left(D_0^*, D_1^*, D_2^*\right) = \int_{-h/2}^{h/2} \frac{E(z,T)}{1-v^2(z,T)}(1,z,z^2)\mathrm{d}z, \quad \left(\overline{D}_0^*, \overline{D}_1^*, \overline{D}_2^*\right) = \int_{-h/2}^{h/2} \frac{v(z,T)E(z,T)}{1-v^2(z,T)}(1,z,z^2)\mathrm{d}z \tag{3b}$$

and

$$D_0 = \frac{D_0^*}{\left(D_0^*\right)^2 - \left(\overline{D}_0^*\right)^2}, \quad \overline{D}_0 = -\frac{\overline{D}_0^*}{\left(D_0^*\right)^2 - \left(\overline{D}_0^*\right)^2} \tag{4a}$$

$$D_1 = \frac{D_0^*D_1^* - \overline{D}_0^*\overline{D}_1^*}{\left(D_0^*\right)^2 - \left(\overline{D}_0^*\right)^2}, \quad \overline{D}_1 = \frac{D_0^*\overline{D}_1^* - \overline{D}_0^*D_1^*}{\left(D_0^*\right)^2 - \left(\overline{D}_0^*\right)^2} \tag{4b}$$

$$D_2 = D_2^* - \frac{D_0^*\left[\left(D_1^*\right)^2 + \left(\overline{D}_1^*\right)^2\right] - 2\overline{D}_0^*D_1^*\overline{D}_1^*}{\left(D_0^*\right)^2 - \left(\overline{D}_0^*\right)^2}, \quad \overline{D}_2 = \overline{D}_2^* - \frac{2D_0^*D_1^*\overline{D}_1^* - \overline{D}_0^*\left[\left(D_1^*\right)^2 + \left(\overline{D}_1^*\right)^2\right]}{\left(D_0^*\right)^2 - \left(\overline{D}_0^*\right)^2} \tag{4c}$$

Based on the Donnell's nonlinear shell theory, the stain-displacement relations defined in the cylindrical coordinate frame can be written as [34]:

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x, \quad \varepsilon_\theta = \varepsilon_\theta^0 + z\kappa_\theta, \quad \gamma_{x\theta} = \gamma_{x\theta}^0 + z\kappa_{x\theta} \tag{5}$$

where ε_x and ε_θ are strain components along x and θ direction, respectively, $\gamma_{x\theta}$ is shear strain in $x\theta$ plane. ε_x^0, ε_θ^0 and $\gamma_{x\theta}^0$ are the membrane strains, defined as: $\varepsilon_x^0 = u_{,x} + w_{,xx}/2$, $\varepsilon_\theta^0 = (v_{,\theta} + w)/R + (w_{,\theta})^2/2R^2$, $\gamma_{x\theta}^0 = v_{,x} + u_{,\theta}/R + w_{,x}w_{,\theta}/R$; while κ_x, κ_θ and $\kappa_{x\theta}$ are the curvatures, given by: $\kappa_x = -w_{,xx}$, $\kappa_\theta = -w_{,\theta\theta}/R^2$ and $\kappa_{x\theta} = -2w_{,x\theta}/R$, where a comma denotes differentiation with respect to x or/and θ variables.

We analyze an infinitely long cylindrical shell, which is unrestricted under any environment temperature. So the constitutive equation does not include explicitly the temperature term, and then the membrane force resultants can be written as

$$\begin{aligned} N_x &= D_0^*\varepsilon_x^0 + \overline{D}_0^*\varepsilon_\theta^0 + D_1^*\kappa_x + \overline{D}_1^*\kappa_\theta \\ N_\theta &= \overline{D}_0^*\varepsilon_x^0 + D_0^*\varepsilon_\theta^0 + \overline{D}_1^*\kappa_x + D_1^*\kappa_\theta \\ 2N_{x\theta} &= \left(D_0^* - \overline{D}_0^*\right)\gamma_{x\theta}^0 + \left(D_1^* - \overline{D}_1^*\right)\kappa_{x\theta} \end{aligned} \tag{6}$$

Reversing Eq. (6), the membrane strains can be expressed as functions of membrane force resultants and curvatures

Table 1
The temperature-dependent properties of metal and ceramic.

Material	Properties	P_{-1}	P_0	P_1	P_2	P_3	P (T = 300 K)
SUS304 (metal)	E (Pa)	0	201.04e9	3.079e−4	−6.534e−7	0	1207.7877e9
	v	0	0.3262	−2.002e−4	3.797e−7	0	0.31776
	ρ (kg/m^3)	0	8166	0	0	0	8166
Si_3N_4 (ceramic)	E (Pa)	0	348.43e9	−3.07e−4	2.160e−7	−8.946e−11	322.2715e9
	v	0	0.24	0	0	0	0.24000
	ρ (kg/m^3)	0	2370	0	0	0	2370