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A variational formulation for dynamic analysis of composite laminated beams based on a general higher-order shear deformation theory

Yegao Qu *, Xinhua Long, Hongguang Li, Guang Meng

State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai 200240, China

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ABSTRACT

This paper presents a general formulation for free and transient vibration analyses of composite laminated beams with arbitrary lay-ups and any boundary conditions. A modified variational principle combined with a multi-segment partitioning technique is employed to derive the formulation based on a general higher-order shear deformation theory. The material couplings of bending-stretching, bending-twist, and stretching-twist as well as the Poisson's effect are taken into account. A considerable number of free and transient vibration solutions are presented for cross- and angle-ply laminated beams with various geometric and material parameters. Different combinations of free, simply-supported, pinned, clamped and elastic-supported boundary conditions are examined. The validity of the formulation is confirmed by comparing the present solutions with analytical and experimental results available in the literature and the ones obtained from finite element analyses. The accuracy of several higher-order shear deformable beam theories for predicting the vibrations of laminated beams has been ascertained. Results of parametric studies for composite beams with different orthotropic ratios, fiber orientations, layer numbers and boundary conditions are also discussed. The present formulation is versatile in the sense that it is capable of accommodating a variety of beam theories available in the literature, and allows the use of different polynomials as admissible functions for composite beams, such as the Chebyshev and Legendre orthogonal polynomials, and the ordinary power polynomials. Moreover, it permits to deal with the linear vibration problems for thin and thick beams subjected to dynamic loads and boundary conditions of arbitrary type.

1. Introduction

Composite laminated beams are extensively used in aircraft structures, space vehicles, turbo-machines and other industrial applications due to their high strength-to-weight and stiffness-to-weight ratios. It is well known that laminated beams in these applications often operate in complex environmental conditions and are commonly exposed to a variety of dynamic excitations which may result in excessive vibration and fatigue damage. A thorough understanding of the vibration behaviors of laminated beams is therefore of particular importance. Despite the many contributions to the analysis of laminated beams, the establishment of reliable and efficient modeling techniques for simulating the dynamic behaviors of generally layered composite beams remains a challenging task and is the focus of the present study.

The development of accurate beam theories has been the subject of significant research interest for many years, and a large amount of beam models have been proposed based on different assumptions and approximations. Excellent overviews of these types of models may be found in Kapania and Raciti [1], Ghugal and Shimpi [2], and Vinson and Sierakowski [3]. Variational principles, including the classical principles (either displacements or stresses as unknowns) and mixed principles (e.g. displacements and stresses simultaneously as unknowns), are usually employed to derive the consistent governing equations and boundary conditions for the theoretical models; see [4–14] for details and proofs. Since a general displacement-based theory will be used in the present study, a brief review related to the displacement-based theories is given below. Physically, laminated beams with general layer-ups are three-dimensional (3D) structures for which the methods of linear elasticity theory may be applied [15]. However, it is well recognized that the solutions of the 3D elasticity equations for composite beams are difficult to obtain and in most cases are even unattainable. Typically, researchers make suitable assumptions concerning the kinematics of deformation or the state of stress through the thickness of the beams, and reduce the 3D beam problems to various 1D representations with reasonable accuracy, such as the equivalent single layer (ESL) model and the layer-wise (LW) model. Following the ordinary classification of

* Corresponding author. Tel.: +86 021 34206332; fax: +86 021 34206814.
E-mail addresses: quyegao@sjtu.edu.cn, quyegao@gmail.edu.cn (Y. Qu).

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the ESL models, there are mainly three major categories, i.e., the classical beam theory (CBT), the first-order beam theory (FBT), and the higher-order beam theory (HBT). The CBT known as Euler–Bernoulli beam theory is the simplest one and is applicable to slender composite beams. For thick beams, the CBT underestimates deflection and overestimates natural frequency due to ignoring the transverse shear deformation effect [1–3]. In order to take into account the effects of transverse shear deformation for the analysis of moderately thick beams, the FBT by Timoshenko has been developed. In this theory, transverse shear strain distribution is assumed to be constant through the beam thickness and, thus, requires a shear correction factor to appropriately represent the strain energy of deformation [1–3,16,17]. It has been shown that the accuracy of the FBT solutions will be strongly dependent on the shear correction factor, and the value of this factor is not a constant but changes with material properties, layer sequences, loading cases, boundary conditions, etc. The limitations of classical beam theory and first-order beam theory stimulated the development of higher-order shear deformation theories to avoid the use of shear correction factors, to include correct cross sectional warping and to get the realistic variation of the transverse shear strains and stresses through the thickness of beam. A number of high-order theories with different shear strain shape functions (including polynomial functions [18–22], trigonometric functions [23–25], exponential functions [26,27], etc.) have been proposed. Although some shape functions mentioned above were initially developed for elastic plates or shells, application of these functions to composite beams is immediate. In the LW models, each layer in a laminated beam is considered to be a separate beam, and compatibility conditions are applied between adjacent layers. These models provide realistic descriptions of kinematics at the ply level and yield accurate stress results for composite laminated beams, but suffer from an excessive number of displacement variables in proportion to the number of layers and hence are not suitable for practical applications, especially when optimization studies are concerned. There also exist some special layer-wise theories, often called zig–zag theories, containing a constant number of unknown variables irrespective of the number of layers in laminated beams. In these theories, the additional unknowns are eliminated by enforcing the continuity of the transverse shear stress components at the interfaces between adjacent layers and by satisfying the zero shear traction conditions on the top and bottom surfaces of the beams. Examples of LW theories are those found in the articles [28–31].

The dynamic analysis of composite laminated beams based on various beam theories has been the subject of significant research activities during the past few years. Early studies have been compiled in the excellent review paper by Kapania and Raciti [32]. In order to properly focus on the features and emphasis of the present paper, a brief review is herein given to the works which are mainly devoted to the free and transient vibration analyses of laminated beams. The dynamic analysis of laminated beams is mostly restricted to free vibrations. A number of analytical and computational methods have been developed and proposed to handle the free vibration problems of laminated beams. They include, but are not limited to, the closed-form solution [3,16,21,22,33,34], variational method [35–39], dynamic stiffness method [40–44], transfer matrix method [45–47], differential quadrature method [48,49], meshless method [50], finite difference method [51–53], and finite element method [54–56]. It should be mentioned that the Poisson's effect, which is often neglected in the one-dimensional laminated beam analysis, has very significant influence on the vibration analysis of composite beams with general layer-ups. The incorporation of this effect involves either correcting the relations of the generalized force/moment resultants and generalized strains of laminated beams or correcting the constitutive equations of a 3D anisotropic body [57]. The transient vibration analysis of composite laminated beams has received less attention compared to the free vibrations. Marur and Kant [58] developed a finite element model with seven degrees of freedom per node for predicting the transient dynamic responses of composite beams. The governing equations of motion were solved using the central difference predictor technique to obtain the response history at different time steps. Sokolinsky and Nutt [59] presented a discretized formulation based on an implicit finite difference method for the time-domain response analysis of sandwich beams. Khdeir [60] investigated the transient vibrations of cross-ply laminated beams by using a generalized modal approach in conjunction with a general higher-order beam theory. Arvin et al. [61] performed a finite element analysis to obtain the structural responses of a composite sandwich beam with viscoelastic core. Kapuria and Alam [62] developed a 1D beam finite element with electric degrees of freedom for the dynamic analysis of hybrid piezoelectric beams by using the layer-wise theory. The Newmark direct time integration method was employed to obtain the transient responses of the composite beams. Tagarielli et al. [63] reported the finite element solutions for the dynamic shock responses of fully clamped monolithic and sandwich beams. Kiral [64] used a three-dimensional finite element model together with the Newmark integration method to obtain the dynamic response of composite beams subjected to moving loads. Mohebpour et al. [65] investigated the dynamic responses of composite laminated beams subjected to a moving oscillator by using the first-order beam theory and finite element method. Based on the mode superposition method, Jafari-Talookolaei et al. [66] studied the dynamic responses of a delaminated beam due to a moving oscillatory mass. Çalim [67] performed a forced vibration analysis of non-uniform composite beams subjected to impulsive loads. The solutions obtained in the Laplace domain were transformed to the time domain by using the Durbin's inverse Laplace transform method. According to the comprehensive survey of the literature, it is found that most of the previous efforts were restricted to laminated beams with limited sets of classical boundary conditions (e.g., the free, simply-supported and clamped edges). Actually, the boundary conditions of a composite beam may not always be classical in engineering applications. This may become one of the main sources of discrepancy when the comparison between theory and experiment is made. Moreover, the existence of various higher-order shear deformable beam theories gives rise to a problem that one may be easily inundated by the abundance of the available models or choices. Although the free vibration results for laminated beams based on some shear deformation theories have been presented and compared by Aydogdu [36,37], to the best of our knowledge, the discrepancies of various higher-order theories in predicting the transient responses of laminated beams have not been investigated. It should be further remarked here that many of the commonly used methods, such as the closed-form method [3,16,21,22,33,34] and the dynamic stiffness method [40–44], are mainly restricted to free vibration analysis; they soon become cumbersome when one wants to deal with the dynamic response problems of composite beams under arbitrary loading cases.

The primary objective of the present investigation is to develop a unified formulation for free and transient vibration analyses of generally layered laminated beams with arbitrary combinations of classical and non-classical boundary conditions. A modified variational principle in conjunction with a multi-segment partitioning technique is employed to derive the formulation based on a general higher-order shear beam theory. The elastic couplings of the bending-stretching, bending-twist and stretching-twist with the Poisson's effect are taken into account. The formulation is particularly attractive since one can choose different polynomials as admissible displacement and rotation functions