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Layerwise solution of free vibrations and buckling of laminated composite and sandwich plates with embedded delaminations

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ABSTRACT

Layerwise plate theory of Reddy, extended for the analysis of delaminations, has served as a basis for development of enriched finite elements. The proposed model assumes layerwise linear variation of in-plane displacements and constant transverse displacement through the plate thickness. Jump discontinuities in displacement field in three orthogonal directions are incorporated using Heaviside step functions, depending on delamination position through the plate thickness. Equations of motion are derived using Hamilton's principle. Using the proposed model laminated composite and sandwich plates were analyzed. All numerical solutions are obtained using originally coded MATLAB programs. Proposed model is verified using existing results from the literature. Results for natural frequencies, mode shapes and critical buckling loads for intact and damaged plates are compared. Effects of plate geometry, lamination scheme, degree of orthotropy and delamination size or position on dynamic characteristics of the plate are presented. Excellent agreement is obtained and a family of new results is presented as a benchmark for future investigations.

1. Introduction

Composite materials attract the increasing attention in the various engineering disciplines. From the materials science point of view, laminated composite plates are usually studied in their micro (fiber) level. In the domain of the theory of plates, of the paramount importance is to describe local (ply level) or global (laminate level) structural response of laminated plate under the various static or dynamic loadings. As the laminated composite plates are used for primary or secondary components of different engineering structures, which are generally loaded with transient loads, it is of great importance to understand the fundamental dynamic characteristics of the laminate, primarily its natural frequencies, mode shapes and critical buckling loads.

Different plate theories have been derived for the structural analysis of composite laminates. Global behavior can be accurately determined by the use of relatively simple equivalent-single-layer laminate theories (ESL), especially for thin laminates with high length-to-thickness ratios. Unfortunately, in the case of the thicker structural components, ESL theories are not adequate, because they neglect shear deformation.ESL theories cannot account for discontinuities in transverse shear strains at the interfaces between layers of different stiffnesses, too. The classical plate theory (CPT) based on the Kirchhoff hypothesis overpredicts natural frequencies and buckling loads due to the neglecting of transverse shear strains [1]. The first-order shear deformation theories (FSDT) are used to take into account the transverse shear effects. The accuracy of obtained results depends on the used shear correction factors [2]. For these reasons there is a need for applying the plate theories of higher (usually third) order. Vuksanović investigated single layer models of higher order (HSDT), which represent plate kinematics with improved accuracy [1]. Global higher-order plate theory is also used for derivation of analytical and numerical solutions for intact cross-ply plates in the work of Matsunaga [3]. Extensive overview of ESL plate theories can be found in the books of Reddy [4] and Staab [5].

Structural members made up of two stiff, strong faces separated by the soft-core are known as sandwich panels. The low weight of sandwich panels was first exploited by the aircraft industry [6]. Of the great importance is the usage of the honey-comb-core sandwich panels, too. In the area of civil engineering, roof and wall panels are used extensively and their core is usually made of foam to provide the thermal isolation of the building, with the low weight. The important part of the analysis of laminated composite and sandwich plates is the structural analysis of the plate with the presence of different forms of damage. Delamination is the most common type of damage for laminated composite plates. It is of the great importance that the bond between the face sheets and soft-core in sandwich plate remain intact for the panel to perform on the appropriate level, so the presence of delamination is of the great danger for the sandwich plates. Due to the presence of these,

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often microscopic, structural defects, loading capacity of the plate is reduced severely [7].

In this paper, Generalized Layerwise Plate Theory of Reddy, extended by Barbero and Reddy [8], is used to analyze the plate response with the presence of delamination. This theory allows independent interpolation of in-plane and out-of-plane displacement components, and also includes possible jump discontinuities at layer interfaces. Piece-wise linear variation of in-plane displacement components, and constant transverse displacement through the thickness are imposed. In the layerwise approach, it is assumed that C^0-continuity through thickness of the laminate is satisfied. It leads that the nodal variables in finite element model are translation components in three orthogonal directions. Cross-sectional warping is taken into account, which is much more kinematically correct representation of displacements [4]. Consistent mass matrix is also employed in the analysis, as shown in work of Hinton et al. [9].

Owen and Li [10] calculated the fundamental frequencies of laminated composite plates using the ESL theories. Refined theory is used in the works of Noor [11,12]. Ju et al. [13] investigated the influence of delamination on mode shapes and natural frequencies using the Mindlin plate theory. The FE model using the layerwise approach is applied in the work of Alnefaie [14]. Ćetković and Vuksanović have derived both the analytical and numerical solution for the intact laminated composite plate using the generalized layerwise plate theory of Reddy [15]. The excellent papers of Yam et al. [16], Lin et al. [17], Wei et al. [18] and Zhen and Wanji [19] served as the benchmark for comparison of the present model with experimental and numerical results. Kim et al. [20] used the through-laminate thickness trigonometric functions to model zigzag in-plane deformation. As shown in Refs. [1,20–22], global higher order ESL theories also overestimate fundamental frequencies for laminated composite and soft-core sandwich plates. This phenomenon is especially pronounced for laminate plates with arbitrary layout [2]. This statement is illustrated and confirmed in this paper, so the layerwise model proposed here is performed for the free vibrations and buckling analysis both for the intact and damaged plates. Zhen and Wanji [19] proposed the global–local higher order theory to overcome the problem of frequency overestimation stated above, while retaining the non-expensive computational effort.

The goal of this work is to present the comparison between fundamental dynamic characteristics of laminated composite and sandwich plates, with or without the presence of embedded delamination. All numerical solutions are obtained using the originally coded MATLAB® program and compared with the existing numerical and experimental data from the literature. Generalized laminated plate theory is used for the derivation of the enriched finite elements with four and nine nodes. Useful programming procedures of different finite element models in MATLAB® can be found in books of Kwon and Bang [23], Ferreira [24] and Voyiadjis and Kattan [25].

2. Formulation of the theory

2.1. Displacement field

We will consider a laminated plate composed of n orthotropic material layers. Global coordinate system is fixed in the mid-plane of the plate, as shown in Fig. 1. Local (material) coordinate system of the each lamina coincides with the fiber direction. N is the number of numerical layers in which nodes through the thickness are located. This number is usually adopted as $N = n + 1$. In the sublaminate concept (see Ref. [4]), more subdivisions (numerical layers) through the thickness of the plate are used for more accurate

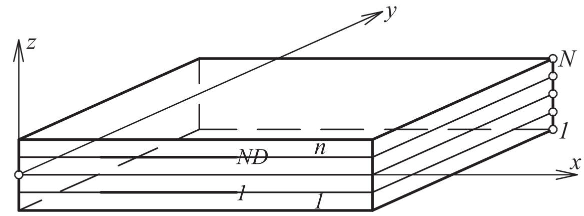

Fig. 1. Typical laminated composite plate in global coordinate system.

interpolation of variables in z-direction. ND is the number of delaminated numerical layers. Overall plate thickness is denoted as h. Lamina thickness is denoted as h_k.

Proposed theory is based on the following assumptions: (1) layers are perfectly bonded together, except in the delaminated area, where jump discontinuities in three orthogonal directions can occur, (2) material is linearly elastic and has three planes of material symmetry, (3) strains are small, (4) geometrical nonlinearity in von Karman sense is included, and (5) inextensibility of normal is imposed. The displacement components (u_1, u_2, u_3) at the arbitrary point (x, y, z) of the laminate can be written as:

$$
\begin{aligned}
u_1(x,y,z) &= u(x,y) + \sum_{I=1}^{N} u^I(x,y)\Phi^I(z) \\
&\quad + \sum_{I=1}^{ND} U^I(x,y)H^I(z) \\
u_2(x,y,z) &= v(x,y) + \sum_{I=1}^{N} v^I(x,y)\Phi^I(z) \\
&\quad + \sum_{I=1}^{ND} V^I(x,y)H^I(z) \\
u_3(x,y,z) &= w(x,y) + \sum_{I=1}^{ND} W^I(x,y)H^I(z)
\end{aligned}
\tag{1}
$$

In Eq. (1), (u, v, w) are displacement components in the middle plane of the laminate, (u^I, v^I) are coefficients to be determined later, and (U^I, V^I, W^I) are jump discontinuities in the displacement field in the Ith delaminated layer. The variable W^I is so-called Crack Opening Displacement (COD), thus the condition $W^I \geqslant 0$ should be adopted to provide no-penetration boundary condition for delaminated surfaces of the I^{th} delamination. This means that if $W^I = 0$, the adjacent layers are in contact condition. The delamination front is the curve in the delamination plane, along which essential boundary conditions $U^I = V^I = W^I = 0$ are enforced. $\Phi^I(z)$ are layerwise continuous functions of z-coordinate. $H^I(z)$ are Heaviside step functions which describe the delamination kinematics in Ith delaminated layer, defined as follows:

$$
H^I(z) = \begin{cases} 1 & z_0 \leqslant z^I \leqslant z \\ -1 & 0 \leqslant z^I < z_0 \end{cases} \quad \frac{dH^I(z)}{dz} = 0 \tag{2}
$$

In the finite element model, (u, v, w) are values of (u_1, u_2, u_3) in the middle plane, (u^I, v^I) are relative values of (u_1, u_2) in relation to the mid-plane in the Ith numerical layer, and (U^I, V^I, W^I) are components of displacement jumps in the Ith delaminated plane. $\Phi^I(z)$ are the one-dimensional Lagrange interpolation functions of thickness coordinate (for the interpolation of in-plane displacements), and $H^I(z)$ are Heaviside step functions as shown in Eq. (2).

In this paper, linear Lagrange interpolation of in-plane displacements is assumed, so in-plane displacements are piece-wise continuous through the laminate thickness in the intact region, and discontinuous in delaminated interfaces. Distribution of the in-plane displacements through the plate thickness is shown in Fig. 2. Cross-sectional warping of line segment AB is shown in