Composite Structures 108 (2014) 77–90

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

On the stability of functionally graded truncated conical shells reinforced by functionally graded stiffeners and surrounded by an elastic medium

Dao Van Dung, Le Kha Hoa *, Nguyen Thi Nga

Vietnam National University, Hanoi, Viet Nam

ARTICLE INFO

Article history:
Available online 13 September 2013

Keywords:
Stiffened truncated conical shell
Stiffener
Elastic medium
Functionally graded material
Critical buckling load

ABSTRACT

This paper presents an analytical approach to investigate the mechanical buckling load of eccentrically stiffened functionally graded truncated conical shells surrounded by elastic medium and subjected to axial compressive load and external uniform pressure. Shells are reinforced by stringers and rings in which material properties of shell and stiffeners are graded in the thickness direction according to a volume fraction power-law distribution. The elastic medium is assumed as two-parameter elastic foundation model proposed by Pasternak. The change of spacing between stringers in the meridional direction is taken into account. The equilibrium and linearized stability equations for stiffened shells are derived based on the classical shell theory and smeared stiffeners technique. The resulting equations which they are the couple set of three variable coefficient partial differential equations in terms of displacement components are investigated by Galerkin method and the closed-form expression for determining the buckling load is obtained. Four cases of stiffener arrangement are analyzed. Carrying out some computations, effects of foundation, stiffener and input factors on stability of shell have been studied. The effectiveness of FGM stiffeners in enhancing the stability of cylindrical shells comparing with homogenuos stiffener is shown.

1. Introduction

Functionally graded shells involving conical shells, in recent years, are widely used in space vehicles, aircrafts, nuclear power plants and many other engineering applications. These structures are usually laid on or placed in a soil medium modeled as an elastic foundation. To increase the resistance of shells to buckling, they are strengthened by stiffeners and thus the critical load can be increased considerably with only a little addition of material. As a results stability and vibration analysis of those strutures are very important problems and have attracted increasing research effort.

In static analysis of conical shells without foundation and stiffener, many studies have been focused on the buckling behavior analysis of shells under mechanic and thermal loading. Seide [1] analyzed the buckling of conical shells under the axial loading. Singer [2] investigated the buckling of conical shells subjected to the axisymmetrical external pressure. Chang and Lu [3] examined the themoelastic buckling of conical shells based on nonlinear analyzes. They used Galerkin method for integrating the equilibrium equation. Tani and Yamaki [4] obtained the results of truncated conical shells under axial compression. Using the Donnell-type shell theory, the linear buckling analysis of laminated conical shells, with orthotropic stretching-bending coupling, under axial compressive load and external pressure, are studied by Tong and Wang [5]. Wu and Chiu [6] studied a three-dimensional solution for the thermal buckling of the laminated composite conical shells.

For shells resting on elastic foundations, many significant results on the vibration and dynamic buckling of isotropic and anitropic cylindrical shells are obtained. Paliwal et al. [7] studied free vibration of circular cylindrical shell on Winkler and Pasternak foundations. Amabili and Dalpiaz [8] investigated free vibration of cylindrical shells with non-asymmetric mass ditribution on elastic bed. Ng and Lam [9] considered effects of elastic foundations on the dynamic stability of cylindrical shells. The same authors [10] analyzed free vibration of rotating circular cylindrical shell on an elastic foundation. Naili and Oddou [11] investigated the buckling of short cylindrical shells surrounded by an elastic medium while Fok [12] studied the buckling of long cylindrical shells embedded in an elastic medium using the energy method. Effects of elastic foundations on the vibration of laminated non-homogenous orthotropic circular cylindrical shells is reported by Sofiyev et al. [13]. Geometrically nonlinear dynamic analysis of doubly curved isotropic shells resting on elastic foundation by a combination of hamonic differential quadrature-finite difference method is presented by Civalek [14]. Solution of axisymmetric dynamic problems for cylindrical shells on an elastic foundation is considered by Golovko et al. [15]. Sheng and Wang [16] considered the effect of thermal load on

* Corresponding author. Tel.: +84 989358315.
E-mail address: lekhahoa@gmail.com (L.K. Hoa).

0263-8223/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2013.09.002

buckling, vibration and dynamic buckling of FGM cylindrical shells embedded in a linear elastic medium based on the first-order shear deformation theory (FSDT) taking into account the rotary inertia and transverse shear strains. The post-buckling analysis of tensionless Pasternak FGM cylindrical shells surrounded by an elastic medium under the lateral pressure and axial load are studied by Shen [17] and Shen et al. [18] using the singular perturbation technique and the higher-order shear deformation shell theory (HDST). The mechanical buckling of FGM cylindrical shells surrounded by Pasternak elastic foundation is studied by Bagherizadeh et al. [19] in which the equilibrium and stability equations are derived based on the higher-order shear deformation shell theory. The stability and vibration analysis of FGM cylindrical shells resting on the Pasternak elastic foundation have been published by Sofiyev et al. [20,21] using Galerkin method to determine buckling load and frequency of shell.

In recent years, because functionally graded material (FGM) conical shells are widely used in modern engineering, so the stability and vibration behaviors of these structures have attracted attention of a lot of scientists. Among those available, Sofiyev [22–24] investigated the linear stability and vibration of un-stiffened FGM truncated conical shells with different boundary conditions. The same author [25] presented the nonlinear buckling behavior and nonlinear vibration [26] of FGM truncated conical shells, and considered [27–31] the buckling of FGM truncated conical shells subjected to axial compressive load and resting on Winkler–Pasternak foundations. For linear analysis, the general characteristics in his works is that the modified Donnell-type equations are used and Galerkin method is applied to obtain closed-form relations of bifurcation type buckling load or to find expressions of fundamental frequencies, whereas for nonlinear analysis, the large deflection theory with von Karman–Donnell-type of kinetic nonlinearity is used. Based on the first-order shell theory by Love–Kirchhoff and the Sanders nonlinear kinetic equations, the thermal and mechanical instability of FGM un-stiffened truncated conical shells also is investigated by Naj et al. [32]. Bich et al. [33] presented results on the buckling of un-stiffened FGM conical panels under mechanical loads. The linearized stability equations in terms of displacement components are derived by using the classical shell theory. Galerkin method is applied to obtained explicit expression of buckling load.

As can be seen that the above introduced works only relate to unstiffened structures. However, in practice, plates and shells including conical shells, usually reinforced by stiffeners system to provide the benefit of added load carrying capability with a relatively small additional weight. Thus, the study on static and dynamic behavior of theses structures are significant practical problem. Weingarten [34] conducted a free vibration analysis for a ring-stiffened simply supported conical shell by considering an equivalent orthotropic shell and using Galerkin method. He also carried out experimental investigations. Crenwelge and Muster [35] applied an energy approach to find the resonant frequencies of simply supported ring-stiffened, and ring and stringer-stiffened conical shells. Mustaffa and Ali [36] studied the free vibration characteristics of stiffened cylindrical and conical shells by applying structural symmetry techniques. Some significant results on vibration of FGM conical shells, cylindrical shells and annular plate structures with a four-parameter power-law distribution based on the first-order shear deformation theory are analyzed by Tornabene [37] and Tornabene et al. [38].

Srinivasan and Krishnan [39] obtained the results on the dynamic response analysis of stiffened conical shell panels in which the effect of eccentricity is taken into account. The integral equation for the space domain and mode superposition for the time domain are used in their work. Based on the Donnell–Mushtari thin shell theory and the stiffeners smeared technique, Mecitoglu [40] studied the vibration characteristics of a stiffened truncated conical shell by the collocation method. The minimum weight design of axially loaded simply supported stiffened conical shells with natural frequency constraints is considered by Rao and Reddy [41]. The influence of placing the stiffeners inside as well as outside the conical shell on the optimum design is studied. The expressions for the critical axial (buckling) load and natural frequency of vibration of conical shell also are derived.

In 2009, Najafizadeh et al. [42] with the linearized stability equations in terms of displacements studied buckling of FGM cylindrical shell reinforced by rings and stringers under axial compression. The stiffeners and skin, in their work, are assumed to be made of functionally graded materials and its properties vary continuously through the thickness direction. Following this direction, Dung and Hoa [43,44] obtained the results on the static nonlinear buckling and post-buckling analysis of eccentrically stiffened FGM circular cylindrical shells under external pressure and torsional load. The material properties of shell and stiffeners are assumed to be continuously graded in the thickness direction. Galerkin method was used to obtain closed-form expressions to determine critical buckling loads. By considering homogenous stiffeners, Bich et al. [45] presented an analytical approach to investigated the nonlinear post-buckling of eccentrically stiffened FGM plates and shallow shells based on the classical shell theory in which the stiffeners are assumed to be homogeneous. Bich et al. [46] obtained the results on the nonlinear dynamic analysis of eccentrically stiffened FGM cylindrical panels. The governing equations of motion were derived by using the smeared stiffeners technique and the classical shell theory with von Karman geometrical nonlinearity. The same authors [47] investigated the nonlinear vibration dynamic buckling of eccentrically stiffened imperfect FGM doubly curved thin shallow shells based on the classical shell theory. The nonlinear critical dynamic buckling load is found according to the Budiansky–Roth criterion. Dung et al. [48] studied a mechanical buckling of eccentrically stiffened functionally graded (ES-FGM) thin truncated conical shells subjected to axial compressive load and uniform external pressure load based on the smeared stiffeners technique and the classical shell theory and considering homogenous stiffeners.

The objective of this study is to extend mechanical buckling results of the work [48] for stiffened FGM thin truncated conical shells surrounded by an elastic medium. The present novelty is that an analytical approach to investigate the buckling load of stiffened functionally graded truncated conical shells surrounded by elastic foundations is presented. Shells under combined load are reinforced by rings and stringers in which their material properties are graded in the thickness direction according to a volume fraction power-law distribution. The change of spacing between stringers in the meridional direction is taken into account. The theoretical formulations based on the smeared stiffeners technique and the classical shell theory, are derived. The resulting equations which they are the couple set of three variable coefficient partial differential equations in terms of displacement components are solved by Galerkin method. The closed-form expressions to determine critical buckling loads are obtained. Four cases of stiffener arrangement are investigated. The influences of various parameters such as foundation, stiffener, dimensional parameters and volume fraction index of materials on the stability of shell are considered. The effectiveness of FGM stiffeners in enhancing the stability of cylindrical shells comparing with homogenuos stiffener is shown.

2. ES-FGM truncated conical shell and derivations

2.1. FGM truncated conical shell

Consider a thin truncated conical shell of thickness h and semi-vertex angle α. The geometry of shell is shown in Fig. 1, where L is