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A *p*-version layerwise model for large deflection of composite plates with curvilinear fibres

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ABSTRACT

A new *p*-version finite element based on a zig-zag layerwise theory is developed. With the proposed model, one can accurately determine the behaviour of laminated composites with different material properties, straight or curvilinear fibres. Comparative studies with published results and with results from finite element software Abaqus are performed to verify the new model. Deflections in the linear and non-linear regimes are then calculated for several constant and variable stiffness composite laminates (VSCL), in which the fibre orientation angle varies linearly in each layer. In a few cases, VSCL plates show better performance in comparison to CSCL plates both in the linear and non-linear regimes. It is also found that it is possible to take advantage of VSCL by varying fibre orientations only in some plies of a laminate, hence reducing manufacturing costs, as well as the amount of gaps and overlaps.

1. Introduction

Fibre reinforced composite materials have been used in a wide range of applications, particularly since the 1980s [1]. These materials offer the possibility to tailor parts, not only by choosing specific materials for matrix and fibres, but also by adjusting the number of plies and the fibre orientation. Of particular interest are lightweight and great strength composite materials, since these characteristics are important in industries such as aerospace and aircraft [1]. More recently, a few studies appeared where it is suggested to enhance the performance of composite materials by varying their stiffness in space [2–16], this type of laminates is known as Variable Stiffness Composite Laminates (VSCL). The variation in stiffness can be achieved by changing the fibre volume fraction in the laminate [2], by dropping or adding plies to the laminate [3], or by using curvilinear fibres [4–16].

We are specifically interested here in laminates reinforced by curvilinear fibres, different aspects of which have been studied in a few papers. Hyer and Lee [4] studied variable stiffness composites with circular holes, by varying the fibre orientations on a region-by-region basis. Tatting and Gürdal [5] manufactured panels by employing a tow-placement machine, to dispose fibres on a curved path within the plane of the laminate. They experimentally studied panels with holes. Moreover, the same authors performed finite element analysis to simulate plates manufactured by a tow-placement machine [6]. Setoodeh et al. [7] tried to find minimum compliance designs by modifying fibre orientation angles and lamination parameters, and minimum compliance designs were obtained for constant and variable stiffness plates. In addition, Setoodeh et al. investigated optimal variable stiffness composite panels for maximum fundamental frequencies in [8], and for maximum buckling load in [9]; in both cases Classical Lamination Theory (CLT) was used. Another study, where curvilinear fibres were employed to maximise natural frequencies, was carried out by Blom et al. [10], who studied variable stiffness conical shells. Camanho et al. [11] carried out buckling and failure analysis of VSCL with cutouts. Although stress concentration in cutouts is the main reason for failure around those regions in traditional Constant Stiffness Composite Laminates (CSCL), they observed that failure in VSCL plates took place in the plates' edges. Their results demonstrate VSCL's advantage over CSCL in redistributing the applied load. Akhavan and Ribeiro [12] studied the natural modes of vibration of VSCL plates with a *p*-version finite element method based upon Third-order Shear Deformation Theory (TSDT). They concluded that using curvilinear fibres meaningfully affects natural frequencies and mode shapes. The TSDT model of [12] was extended to take into account geometrical non-linearity in [13], where static analyses were performed. Forced, non-linear vibrations of variable stiffness composite plates were studied in the time domain by employing a First-order Shear Deformation Theory (FSDT) model in [14]. In [15] a similar model is employed to investigate the evolution of the modes of vibration of VSCL with the vibration amplitude. These studies in the non-linear regime, [14,15], showed that small variations in the fibre path may lead to quite diverse oscillations, where, for example, different modes

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appear in the response. Dang and Hallett [16] studied the behaviour of VSCLs on impact and post-impact, and concluded that material orientation plays an important role on the evolution of delaminations in impact and post-impact.

According to the literature, curvilinear fibres can have a substantial effect in the structural behaviour of laminates, influencing the natural frequencies and mode shapes of vibration, the response to external loads, critical buckling loads or failure resistance. Studies on VSCL often follow Equivalent Single Layer (ESL) theories and, to the best of the authors' knowledge, there is no layerwise finite element specifically developed for VSCL with curvilinear fibres. Although ESL theories can provide a good estimate of the overall mechanical behaviour, they are not adequate when one wishes a more detailed, localised, picture or when cross sectional warping is important, which occurs particularly in thicker laminates. In layerwise theories each layer is considered separately, continuity of displacements is imposed at the interfaces, but the derivatives of displacement components can be discontinuous at a number of layers through the thickness, providing for a more correct consideration of cross sectional warping than ESL [17]. Detailed description of plies in the laminates is performed by laywerwise approaches, which can be based on CLT, FSDT or a Higher Order Theory (HOT) at a layer level [18]. This method was applied by Bert et al. [19,20] to analyse static bending and vibrations of laminated composite plates. The stresses and deflections obtained were in good agreement with results by three-dimensional elasticity theory. More vibration modes are predicted by this theory as well. Moreira et al. [21,22] assumed a linear variation of the in-plane displacements in each layer, imposing displacement continuities at the interfaces, to implement a layerwise theory and a zig-zag formulation.

It is worth noting that in this paper the p-version finite element method is applied. In Ref. [23], it is proved that the p-version FEM outperforms – chiefly in what the rate of convergence is concerned, although it also has other advantages – the h-version FE in smooth linear problems. There are several publications addressing linear and non-linear problems, where numerical tests indicate that p-version finite elements possess a fast convergence rate [24–28]. Interestingly, there appears to be no laywerwise p-version element available.

In this paper, a new p-version finite element, with hierarchic basis functions and based on zig-zag layerwise theory is proposed to investigate the non-linear behaviour of variable stiffness composite plates with curvilinear fibres. Newton–Raphson's method is used to solve the equations of equilibrium. By using the proposed model, the behaviour of laminated plates with various material properties and curvilinear fibre orientations can accurately be determined. A comparative study with published results and with results from Abaqus finite element software [29] is performed to verify the accuracy of the new model. Linear and non-linear deflections of constant and variable stiffness composites are obtained. Finally, studies on the behaviour of constant stiffness composite plates, variable stiffness composites only with curved fibres, and plates which simultaneously take advantage of straight and curvilinear fibres are carried out.

2. Formulation

The model adopted here is based on a zig-zag layerwise theory [21,22] and on a p-version finite element approach [23]. The selected theory is capable to predict the behaviour of composite plates with different material properties and fibre orientations, including curvilinear fibres. It represents an advance in comparison to the p-elements of [12–15], where equivalent single layer approaches were adopted for VSCL, since it provides for a better description of the variation of in-plane displacements through the thickness and therefore of cross-sectional warping. The former aspects are in particular important for thick laminates or when a detailed description of strains (or stresses) is intended. In this method, instead of using an equivalent single layer, an N-layer laminated composite is in fact represented by several layers (the number of which can or not be N), each with its own in-plane displacement variables. The displacement continuity at layers interfaces is imposed, but continuity of displacement derivatives with respect to the spatial coordinates is not. Fig. 1 shows a schematic view of the displacement fields and coordinate systems. Each layer has its own reference axes (x_k, y_k, z_k), parallel to the respective axes of a Cartesian coordinate system of reference (x, y, z). Whilst coordinate z_k represents a translation of the z axis, coordinates x_k and x are equal, as well as y_k and y; so we will write the displacement field as a function of (x, y, z_k).

Within each VSCL layer, a linear variation of fibre angle is considered, so that the fibres look like it is shown in Fig. 1. In relation to the global coordinate system, also shown in Fig. 1, and by assuming T_0 as the fibre angle at (0, 0) and T_1 as the fibre angle at the panel's edges, the fibre orientation is defined by:

$$\theta^k(x) = \frac{2\left(T_1^k - T_0^k\right)}{a}|x| + T_0^k \tag{1}$$

2.1. Displacement field

Displacement components in the x, y and z_k directions of layer kth of the model are defined by U^k, V^k and W^k, respectively. The following displacement field, based on first-order shear deformation theory, is assumed for each individual layer:

$$\mathbf{R}^k = \left\{ \begin{array}{l} U^k(x,y,z_k) \\ V^k(x,y,z_k) \\ W^k(x,y,z_k) \end{array} \right\} = \left\{ \begin{array}{l} \frac{u^k(x,y)}{2} + \frac{u^{k+1}(x,y)}{2} + \frac{u^{k+1}(x,y) - u^k(x,y)}{h_k} z_k \\ \frac{v^k(x,y)}{2} + \frac{v^{k+1}(x,y)}{2} + \frac{v^{k+1}(x,y) - v^k(x,y)}{h_k} z_k \\ w(x,y) \end{array} \right\} \tag{2}$$

where subscript k indicates layer number, $u^k(x,y)$ and $v^k(x,y)$ are displacements components at the bottom surface, $u^{k+1}(x,y)$ and $v^{k+1}(x,y)$ are displacements at the top surface of layer k, and $w(x,y)$ is the transverse displacement, assumed to be independent from z. The thickness of layer k is represented by h_k; the thickness of the plate is h. As already mentioned, in the layerwise theory here employed each layer is considered as a separate plate and the

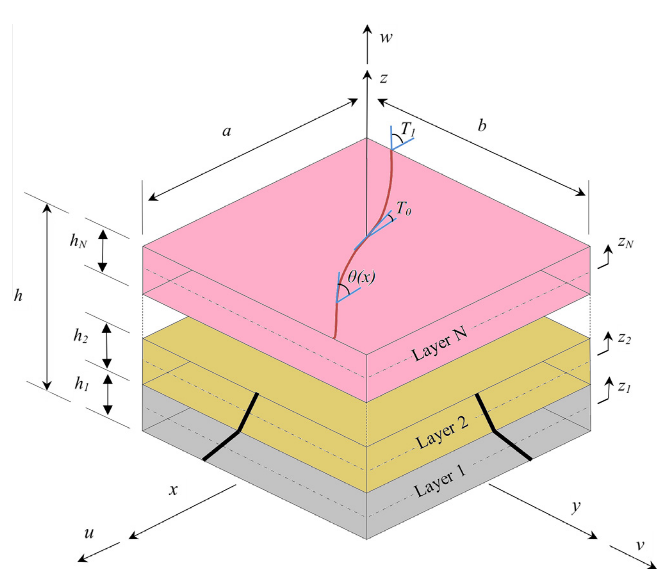

Fig. 1. Schematic view of displacement fields and curvilinear fibre.