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On an exact bending curvature model for nonlinear free vibration analysis shear deformable anisotropic laminated beams

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ABSTRACT

Geometrically nonlinear free vibration analysis of shear deformable anisotropic laminated composite beams resting on a two-parameter elastic foundation is presented. The material of each layer of the beam is assumed to be linearly elastic and fiber-reinforced. A new nonlinear beam model involving the exact expression of bending curvature is introduced, and the nonlinear vibration analysis with exact nonlinear characteristics of the work done by axial loading is accordingly performed. The governing equations are based on higher order shear deformation beam theory with a von Kármán-type of kinematic nonlinearity and including the bending–stretching, bending–twisting, and stretching–twisting couplings. Two kinds of end conditions, namely movable and immovable, are considered, and a perturbation technique is employed to determine the linear and nonlinear frequencies of a composite beam with or without initial stresses. The frequency response of laminated beams with different geometric and material parameters, end conditions and effect on elastic foundation is numerically illustrated. The results reveal that the geometric and physical properties, end conditions, and elastic foundation effect have a significant influence on large amplitude vibration behavior of anisotropic laminated composite beams.

1. Introduction

Composite beam structures are widely used in various engineering applications (e.g., airplane wings, helicopter blades as well as many others in the aerospace, mechanical, and civil industries). Anisotropic composites provide more design flexibility than conventional materials. Due to the outstanding engineering properties, such as high strength/stiffness to weight ratios, the laminated composite beams are likely to play a remarkable role in design of various engineering type structures and partially replace the conventional isotropic beam structures. However, their increased amount of design parameters brings some difficulties in structural analysis. One of the important problems in engineering structures is the vibration analysis of composite beams.

A number of useful elastic beams, plates and shells theories have been proposed for the analysis of composite structures [1–10] due to the rapid increasing use of advanced composite materials in various industries. The practical importance and potential benefits of the composite beams have inspired continuing research interest. A series of research work in the vibration analysis of composite beams have been conducted in the past decades. Excellent overviews of these types of models may be found in Kapania and Raciti [11], Rose [12], and Hajianmaleki and Qatu [13]. Many analytical and computational methods including, but being not limited to, the closed-form solution [14–20], dynamic stiffness method [21–30], differential quadrature method (DQM) [31–38], finite element method (FEM) [39–42] and finite difference method [43–45], have been developed to deal with linear and nonlinear vibrations of laminated beams. Linear free vibration analysis of composite beams on an elastic foundation has received a good amount of attention in the literature [14–18,21–31,34–37,39–48]; however, relatively few studies have been devoted for nonlinear vibration aspects of such beams [19,20,32,33,38,49]. Furthermore, Aydogdu [50,51], Wu and Chen [52] and Qu et al. [53] presented and compared a lot of free vibration results for laminated beams using some of shear deformation-based theories, and the results showed that the natural frequencies obtained from different higher order theories in predicting the free vibration of laminated beams become unremarkable. It should be given more attention to the elastic coupling of the bending–stretching, bending–twisting and stretching–twisting resulted from the anisotropic nature of material. On the other hand, the laminated beams often undergo large amplitude vibrations when they are subjected to severe dynamic loading, and the nonlinear free vibration analysis of composite beams have aroused interest of researchers. Ganapathi et al. [54] presented the large amplitude free vibration analysis of cross-ply laminated straight and curved beams using

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the spline element method. Singh et al. [55] studied the large-amplitude free vibrations of unsymmetrically laminated beam using von-Kármán large deflection theory using one-dimensional finite elements based on classical lamination theory, first-order shear-deformation theory, and higher-order shear-deformation theory. Patel et al. [56] studied free vibration and post-buckling analysis of laminated orthotropic beams resting on a two parameters elastic foundation (Pasternak type) using a three-node shear flexible beam element. Recently, Gunda et al. [19] investigated large amplitude vibration analysis of laminated composite beam with axially immovable ends with symmetric and asymmetric lay-up orientations by using the Rayleigh–Ritz (R–R) method. They solved and evaluated the nonlinear governing equation for the large amplitude vibration of the composite beam with the nonlinear finite element formulation. In fact, in spite of the availability of finite element method and powerful computer programs, the second- or higher-order analysis of a composite beam is still an impractical task to most structural designers due to the limitation of the number of degrees of freedom (DOFs) required to achieve a desired level of precision and efficiency. To the authors' best knowledge there is no work available in the literature on the large amplitude free flexural vibration analysis of shear deformable anisotropic laminated composite beams with or without initial stress on two-parameter nonlinear elastic foundation using the two-step perturbation method.

For nonlinear analysis of beams, the key issue is how to conduct the nonlinear model in the governing equations. In fact, the beam is assumed to undergo moderate rotations and only the nonlinear terms resulting from the development, up to a maximum of the third order, of generalized strains in terms of the beam cross-section rotation are retained. There are two approaches used in the previous studies. In the first approach, the nonlinear model is based on the exact expression of curvature $\frac{\partial\theta}{\partial X}$ ($\theta$is the slope of the deflected beam); while in the second approach, the linear expression of curvature $\frac{\partial^2\overline{W}}{\partial X^2}$ remains and the von Kármán-type strain–displacement relation of the beam in the longitudinal direction is introduced. The major difference between these two approaches lies in that the nonlinear term in the second approach depends on the extensional rigidity and it does not appear in the beam large amplitude vibrations. However, different from that of plate and shell structures, the high order term of the exact expression of curvature, when compared to that of the axial strain–displacement relation of the beams, may play a much more significant role in geometrically nonlinear analysis of beams. Although the above described approaches were successful in predicting the linear frequencies in a variety of beams with various end conditions and for situations where geometrically nonlinear effects were important, the approaches can become complex and perhaps difficult to study nonlinear vibration of a beam due to geometrically nonlinear effect, which requires an exact analysis of the curvature. When deriving beam models for the moderate rotational range of deformation, special attention has to be given to the truncation order as well as the used physical assumptions. On the other hand, the sufficient higher order terms are needed to investigate the beam deformation range of moderate rotations as the order of truncations of strains strongly affects the results.

The present work thus focuses on nonlinear free vibration behavior of anisotropic laminated composite beams resting on two-parameter (Pasternak-type or Vlasov-type) elastic foundations. The governing equations by applying Hamilton's principle are based on the higher order shear deformation beam theory with a von Kármán-type of kinematic nonlinearity including beam-foundation interaction and introducing an exact bending curvature model (the work V done by the axial loading of laminated composite beam is exactly expressed and adopted in the present nonlinear analysis). For nonlinear vibration problems, two kinds of the end boundary conditions, i.e., movable and immovable, are considered. The analysis uses a two-step perturbation technique to determine the nonlinear frequencies of a beam with or without initial stresses. The numerical illustrations concern the nonlinear vibration behavior of anisotropic laminated composite beams with different types of geometric parameters and ply arrangements (layups) of shear deformable anisotropic laminated composite beams.

2. Theoretical development

Consider a laminated composite beam with width b, length L and thickness h, which consists of N plies of any kind. The beam resting on a two-parameter elastic foundation is assumed to be relatively thick, and is subjected to the axial compressive loading P. The axial and transverse displacement fields are expressed as

$$\overline{U}_1(X,Y,Z,t) = \overline{U}(X,t) + Z\overline{\Psi}(X,t) - c_1Z^3\left(\overline{\Psi} + \frac{\partial\overline{W}}{\partial X}\right) \tag{1a}$$

$$\overline{U}_2(X,Y,Z,t) = 0 \tag{1b}$$

$$\overline{U}_3(X,Y,Z,t) = \overline{W}(X,t) \tag{1c}$$

where $\overline{U}_1$and $\overline{U}_3$ are the displacements in X- and Z-directions at any material point in the (X,Z) plane, respectively. $\overline{U}$ and $\overline{W}$ are the longitudinal and transverse displacements along the beam reference plane (X,Y), and $\overline{\Psi}$ is the rotation of the normal to the cross-section about Y-axis at the reference plane. The bending curvature is modified to the form of $\frac{\partial^2\overline{W}}{\partial X^2}\left[1-\left(\frac{\partial\overline{W}}{\partial X}\right)^2\right]^{-1/2}$ to replace $\frac{\partial^2\overline{W}}{\partial X^2}$. Z is the depth of the material point measured from the beam reference plane along the positive Z-axis. The strains can be written as

$$\{\varepsilon_X\} = \left\{\varepsilon_X^{(0)}\right\} + Z\left\{\varepsilon_X^{(1)}\right\} + Z^3\left\{\varepsilon_X^{(3)}\right\} \tag{2}$$

$$\{\gamma_{ZX}\} = \left\{\gamma_{ZX}^{(0)}\right\} + Z^2\left\{\gamma_{ZX}^{(2)}\right\} \tag{3}$$

where

$$\{\boldsymbol{\varepsilon}^{(0)}\} = \left\{\varepsilon_X^{(0)}\right\} = \left\{\frac{\partial\overline{U}}{\partial X} + \frac{1}{2}\left(\frac{\partial\overline{W}}{\partial X}\right)^2\right\},\ \{\boldsymbol{\varepsilon}^{(1)}\} = \left\{\varepsilon_X^{(1)}\right\} = \left\{\frac{\partial\overline{\Psi}}{\partial X}\right\} \tag{4a}$$

$$\{\boldsymbol{\varepsilon}^{(3)}\} = \left\{\varepsilon_X^{(3)}\right\} = -c_1\left\{\frac{\partial\overline{\Psi}}{\partial X} + \frac{\partial^2\overline{W}}{\partial X^2}\left[1-\left(\frac{\partial\overline{W}}{\partial X}\right)^2\right]^{-1/2}\right\} \tag{4b}$$

$$\{\boldsymbol{\gamma}^{(0)}\} = \left\{\gamma_{ZX}^{(0)}\right\} = \left\{\overline{\Psi} + \frac{\partial\overline{W}}{\partial X}\right\},\ \{\boldsymbol{\gamma}^{(2)}\} = \left\{\gamma_{ZX}^{(2)}\right\} = -3c_1\left\{\left(\overline{\Psi} + \frac{\partial\overline{W}}{\partial X}\right)\right\} \tag{4c}$$

The laminated plate constitutive equations based on the higher-order shear deformation theory can be expressed as

$$\begin{Bmatrix} \overline{N}_X \\ \overline{N}_Y \\ \overline{N}_{XY} \\ \overline{M}_X \\ \overline{M}_Y \\ \overline{M}_{XY} \\ \overline{P}_X \\ \overline{P}_Y \\ \overline{P}_{XY} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & E_{11} & E_{12} & E_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & E_{12} & E_{22} & E_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & E_{16} & E_{26} & E_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & F_{11} & F_{12} & F_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & F_{12} & F_{22} & F_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & F_{16} & F_{26} & F_{66} \\ E_{11} & E_{12} & E_{16} & F_{11} & F_{12} & F_{16} & H_{11} & H_{12} & H_{16} \\ E_{12} & E_{22} & E_{26} & F_{12} & F_{22} & F_{26} & H_{12} & H_{22} & H_{26} \\ E_{16} & E_{26} & E_{66} & F_{16} & F_{26} & F_{66} & H_{16} & H_{26} & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_X^{(0)} \\ \varepsilon_Y^{(0)} \\ \gamma_{XY}^{(0)} \\ \varepsilon_X^{(1)} \\ \varepsilon_Y^{(1)} \\ \gamma_{XY}^{(1)} \\ \varepsilon_X^{(3)} \\ \varepsilon_Y^{(3)} \\ \gamma_{XY}^{(3)} \end{Bmatrix} \tag{5}$$

where $\overline{N}_X$ and $\overline{M}_X$ are the stretching force resultant and bending moment resultant, respectively; $\overline{P}_X$ represents the high-order bending moment resultant. Imposing the traction boundary conditions at the top and bottom free-surfaces ($Z = \pm h/2$), the transverse shear stress will vanish, i.e., $\tau_{ZX} = 0$, and the coefficient c_1 in the displacement field Eq. (1a) is $c_1 = 4/3h^2$.