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Wavelet spectral finite element for wave propagation in shear deformable laminated composite plates

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ABSTRACT

This paper presents a new 2-D wavelet spectral finite element (WSFE) model for studying wave propagation in thin to moderately thick anisotropic composite laminates. The WSFE formulation is based on the first order shear deformation theory (FSDT) which yields accurate results for wave motion at high frequencies. The wave equations are reduced to ordinary differential equations (ODEs) using Daubechies compactly supported, orthonormal, scaling functions for approximations in time and one spatial dimension. The ODEs are decoupled through an eigenvalue analysis and then solved exactly to obtain the shape functions used in element formulation. The developed spectral element captures the exact inertial distribution, hence a single element is sufficient to model a laminate of any dimension in the absence of discontinuities. The 2-D WSFE model is highly efficient computationally and provides a direct relationship between system input and output in the frequency domain. Results for axial and transverse wave propagations in laminated composite plates of various configurations show excellent agreement with finite element simulations using Abaqus®.

1. Introduction

Wave propagation in elastic structures has been studied extensively and applied for transient response prediction, mechanical property characterization, and nondestructive evaluation (NDE) [1–5]. Composite (elastic) structures are increasingly used in many industries such as transportation (air, land, and sea), wind energy, and civil infrastructure due to several advantages including higher specific strength and modulus, fewer joints, improved fatigue life, and higher resistance to corrosion. Lamb wave based structural health monitoring (SHM), which aims to perform nondestructive evaluation through integrated actuators and sensors, has been a very active area of research in the past decade [6–10]. A validated physics-based model for wave propagation combined with experimental measurements is generally required for complete characterization (presence, location, and severity) of damages.

The modeling of wave propagation in composites presents complexities beyond that for isotropic structures [2,4]. Analytical solutions for wave propagation are not available for most practical structures due to complex nature of governing differential equations and boundary/initial conditions. The finite element method (FEM) is the most popular numerical technique for modeling wave propagation phenomena. However, for accurate predictions using FEM, typically 20 elements should span a wavelength [11], which results in very large system size and enormous computational cost for wave propagation analysis at high frequencies. In addition, solving inverse problems (as required for NDE/SHM) is very difficult using FEM. Spectral finite element (SFE), which follows FEM modeling procedure in the transformed frequency domain, is highly suitable for wave propagation analysis [12–14]. SFE models are many orders smaller than FEM and highly suitable for efficient NDE/SHM. Frequency domain formulation of SFE enables direct relationship between output and input through system transfer function (frequency response function). SFE has very high computational efficiency since nodal displacements are related to nodal tractions through frequency-wave number dependent stiffness matrix. Mass distribution is captured exactly and the accurate elemental dynamic stiffness matrix is derived. Consequently, in the absence of any discontinuities, one element is sufficient to model a beam or plate structure of any length.

Fast Fourier Transform (FFT) based Spectral Finite Element (FSFE) method was popularized by Doyle [12], who formulated FSFE models for isotropic 1-D and 2-D waveguides including elementary rod, Euler Bernoulli beam, and thin plate. Gopalakrishnan and associates [13,15] extensively investigated FSFE models for beams and plates-with anisotropic and inhomogeneous material properties. The FSFE method is very efficient for wave motion analysis and it is suitable for solving inverse problems; however, FSFE

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cannot model waveguides of short lengths. For 2-D problems, FSFE are essentially semi-infinite, that is, they are bounded only in one direction [12,13]. Due to the global basis functions of the Fourier series approximation of the spatial dimension, the effect of lateral boundaries cannot be captured. In addition, FSFE requires assumption of periodicity in time approximation resulting in "wrap-around" problem for smaller time window, which totally distorts the response.

The 2-D Wavelet based Spectral Finite Element (WSFE) presented by Gopalakrishnan and Mitra [14] overcomes the "wrap-around" problem and can accurately model 2-D plate structures of finite dimensions. WSFE uses orthogonal compactly supported (localized) Daubechies scaling functions [16] as basis for both temporal and spatial approximations. Gopalakrishnan and associates have formulated WSFE elements for wave propagation in rods, higher order beams, and plates with both isotropic and anisotropic material properties [17–20]. However, the 2-D WSFE plate formulation presented in [14,19,20] is based on the classical laminated plate theory (CLPT) [21]. The CLPT based formulations exclude transverse shear deformation and rotary inertia resulting in significant errors for wave motion analysis at high frequencies, especially for composite laminates which have relatively low transverse shear modulus [22,23]. Wave propagation in composite laminates based on the first order shear deformation theory (FSDT) [21], which accounts for transverse shear and rotary inertia, yields accurately results comparable with 3-D elasticity solutions and experiments even at high frequencies [22,23]. For isotropic materials FSDT is known to be exceptionally accurate down to wavelengths comparable with the plate thickness h, whereas CLPT is of acceptable accuracy only for wavelengths greater than, say, $20h$ [24].

This paper presents a new 2-D WSFE based on FSDT for high frequency analysis of waveguides with finite dimensions and anisotropic material properties. Governing partial differential equations (PDEs) for wave motion and their temporal approximation using Daubechies compactly supported high-order scaling functions are presented. An eigenvalue analysis is performed to decouple the reduced PDEs in spatial dimensions. The decoupled PDEs are then approximated in one spatial dimension using Daubechies lower-order scaling functions followed by an eigenvalue analysis similar to the time approximation. The resulting ordinary differential equations (ODEs) are solved exactly in frequency-wavenumber domain and the solution is used as shape function for the 2-D spectral element. Numerical experiments are performed to highlight the differences between FSDT and CLPT in dispersion curves, provide spectrum relationships, and present time domain responses. Results for the new WSFE formulation are validated with Abaqus® simulations using shear flexible shell elements [25].

2. Formulation of wavelet spectral finite element with shear deformation

Two dimensional wavelet spectral finite element with shear deformation is formulated here for anisotropic composite laminates.

2.1. Governing differential equations for wave propagation

Consider a laminated composite plate of thickness h with the origin of the global coordinate system at the mid-plane of the plate and Z axis being normal to the mid-plane as shown in Fig. 1(a). Fig. 1(b) shows the corresponding nodal representation with degrees of freedom (DOFs). Using FSDT, the governing partial differential equations (PDEs) for wave propagation have five degrees of freedom: u, v, w, ϕ, and, ψ. The terms $u(x,y,t)$ and $v(x,y,t)$ are mid-plane ($z = 0$) displacements along X and Y axes; $w(x,y,t)$ is transverse displacement in Z direction, and $\psi(x,y,t)$ and $\phi(x,y,t)$ are the rotational displacements about X and Y axes, respectively. The displacements w, ϕ and ψ do not change along the thickness (Z direction). The quantities (N_{xx},N_{xy},N_{yy}) are in-plane force resultants, (M_{xx},M_{xy},M_{yy}) are moment resultants, and (Q_x,Q_y) denote the transverse force resultants.

The FSDT displacement field Eq. (1) represents a constant shear strain in the transverse (thickness) direction.

$$\begin{aligned} U(x,y,z,t) &= u(x,y,t) + z\phi(x,y,t) \\ V(x,y,z,t) &= v(x,y,t) + z\psi(x,y,t) \\ W(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{1}$$

The equations of motion based on the above displacement field are given by [21],

$$\begin{aligned}
&A_{11}\frac{\partial^2 u}{\partial x^2} + 2A_{16}\frac{\partial^2 u}{\partial x\partial y} + A_{66}\frac{\partial^2 u}{\partial y^2} + A_{16}\frac{\partial^2 v}{\partial x^2} + (A_{12}+A_{66})\frac{\partial^2 v}{\partial y\partial x} \\
&\quad + A_{26}\frac{\partial^2 v}{\partial y^2} + B_{11}\frac{\partial^2 \phi}{\partial x^2} + 2B_{16}\frac{\partial^2 \phi}{\partial x\partial y} + B_{66}\frac{\partial^2 \phi}{\partial y^2} + B_{16}\frac{\partial^2 \psi}{\partial x^2} \\
&\quad + (B_{12}+B_{66})\frac{\partial^2 \psi}{\partial y\partial x} + B_{26}\frac{\partial^2 \psi}{\partial y^2} = I_0\frac{\partial^2 u}{\partial t^2} + I_1\frac{\partial^2 \phi}{\partial t^2} \\
&A_{16}\frac{\partial^2 u}{\partial x^2} + (A_{12}+A_{66})\frac{\partial^2 u}{\partial y\partial x} + A_{26}\frac{\partial^2 u}{\partial y^2} + A_{66}\frac{\partial^2 v}{\partial x^2} + 2A_{26}\frac{\partial^2 v}{\partial x\partial y} \\
&\quad + A_{22}\frac{\partial^2 v}{\partial y^2} + B_{16}\frac{\partial^2 \phi}{\partial x^2} + (B_{12}+B_{66})\frac{\partial^2 \phi}{\partial y\partial x} + B_{26}\frac{\partial^2 \phi}{\partial y^2} + B_{66}\frac{\partial^2 \psi}{\partial x^2} \\
&\quad + 2B_{26}\frac{\partial^2 \psi}{\partial x\partial y} + B_{22}\frac{\partial^2 \psi}{\partial y^2} = I_0\frac{\partial^2 v}{\partial t^2} + I_1\frac{\partial^2 \psi}{\partial t^2} \\
&KA_{55}(\frac{\partial \phi}{\partial x} + \frac{\partial^2 w}{\partial x^2}) + KA_{45}(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial y\partial x}) + KA_{44}(\frac{\partial \psi}{\partial y} + \frac{\partial^2 w}{\partial y^2}) \\
&\quad = I_0\frac{\partial^2 w}{\partial t^2} \\
&B_{11}\frac{\partial^2 u}{\partial x^2} + 2B_{16}\frac{\partial^2 u}{\partial y\partial x} + B_{66}\frac{\partial^2 u}{\partial y^2} + B_{16}\frac{\partial^2 v}{\partial x^2} + (B_{12}+B_{66})\frac{\partial^2 v}{\partial y\partial x} \\
&\quad + B_{26}\frac{\partial^2 v}{\partial y^2} + D_{11}\frac{\partial^2 \phi}{\partial x^2} + 2D_{16}\frac{\partial^2 \phi}{\partial y\partial x} + D_{66}\frac{\partial^2 \phi}{\partial y^2} + D_{16}\frac{\partial^2 \psi}{\partial x^2} \\
&\quad + (D_{12}+D_{66})\frac{\partial^2 \psi}{\partial y\partial x} + D_{26}\frac{\partial^2 \psi}{\partial y^2} - KA_{55}(\phi + \frac{\partial w}{\partial x}) - KA_{45}(\psi + \frac{\partial w}{\partial y}) \\
&\quad = I_1\frac{\partial^2 u}{\partial t^2} + I_2\frac{\partial^2 \phi}{\partial t^2} \\
&B_{16}\frac{\partial^2 u}{\partial x^2} + (B_{12}+B_{66})\frac{\partial^2 u}{\partial y\partial x} + B_{26}\frac{\partial^2 u}{\partial y^2} + B_{66}\frac{\partial^2 v}{\partial x^2} + 2B_{26}\frac{\partial^2 v}{\partial y\partial x} \\
&\quad + B_{22}\frac{\partial^2 v}{\partial y^2} + D_{16}\frac{\partial^2 \phi}{\partial x^2} + (D_{12}+D_{66})\frac{\partial^2 \phi}{\partial y\partial x} + D_{26}\frac{\partial^2 \phi}{\partial y^2} + D_{66}\frac{\partial^2 \psi}{\partial x^2} \\
&\quad + 2D_{26}\frac{\partial^2 \psi}{\partial y\partial x} + D_{22}\frac{\partial^2 \psi}{\partial y^2} - KA_{45}(\phi + \frac{\partial w}{\partial x}) - KA_{44}(\psi + \frac{\partial w}{\partial y}) \\
&\quad = I_1\frac{\partial^2 v}{\partial t^2} + I_2\frac{\partial^2 \psi}{\partial t^2}
\end{aligned} \tag{2}$$

where the stiffness constants A_{ij}, B_{ij}, D_{ij} and the inertial coefficients I_0, I_1 and I_2 are defined as

$$\begin{aligned} [A_{ij}, B_{ij}, D_{ij}] &= \sum_{q=1}^{N_p}\int_{z_q}^{z_{q+1}} \overline{Q}_{ij}[1, z, z^2]dz, \quad \{I_0, I_1, I_2\} \\ &= \int_{-h/2}^{h/2}\{1, z, z^2\}\rho dz \end{aligned} \tag{3}$$

The term $\overline{Q}_{ij}$ are the stiffnesses of the q^{th} lamina in laminate coordinate system, N_p is the total number of laminae (plies), ρ is the mass