Composite Structures 108 (2014) 435–448

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

The Haar wavelet method for free vibration analysis of functionally graded cylindrical shells based on the shear deformation theory

Guoyong Jin *, Xiang Xie, Zhigang Liu

College of Power and Energy Engineering, Harbin Engineering University, Harbin 150001, PR China

ARTICLE INFO

Article history:
Available online 28 September 2013

Keywords:
Haar wavelet series
FG cylindrical shell
Free vibrations

ABSTRACT

A simple yet efficient solution approach based on the Haar wavelet is presented for the free vibration analysis of functionally graded (FG) cylindrical shell. The first-order shear deformation shell theory is adopted to formulate the theoretical model. The separation of variables is first performed according to Levi approach; then Haar wavelet discretization is applied with respect to the axial direction and Fourier series is assumed with respect to the circumferential direction. The constants appearing from the integrating process are determined by boundary conditions, and thus the partial differential equations are transformed into algebraic equations. The natural frequencies of the FG cylindrical shells are obtained by solving algebraic equations. The accuracy and reliability of the current solutions are validated by numerical examples and comparison with the results available in the literature. It is shown that accurate frequencies can be obtained by using a small number of collocation points and boundary conditions can be easily achieved. Detailed parametric analysis is carried out to show the effects of some geometrical and material parameters on the natural frequencies of FG cylindrical shells. The advantages of this current solution method consist in its simplicity, fast convergence, low computational cost and high precision.

1. Introduction

With the development of modern industries, a new class of composite materials known as functionally graded materials (FGMs) has drawn considerable attention. The term FGMs was originated in the mid-1980s by a group of scientists in Japan. Since then, an effort to develop high-resistant materials using FGMs had been continued. However, the analysis of FGMs structures is more complicated than that of homogeneous material structures, owing to the spatial variations of material properties. In order to investigate the stress and displacement fields of FGMs body, one needs to solve the partial differential equations with variable coefficients. This class of problem is challenging and thus there is a lot of literature on the mechanical behavior of FGMs structures. As one of important and common structural components, cylindrical shells of composite materials are widely used in practical engineering applications. Therefore, vibration analysis of FG cylindrical shell often is required and of great technical importance, allowing the designers and engineers to provide optimal design and avoid unpleasant, inefficient and structurally damaging resonant.

The increasing use of composite shell structures has motivated great interest in developing various shell theories and computational approaches for analyzing their dynamic behaviors [1–28]. Especially in the last several decades, a huge amount of research efforts have been devoted to the vibration analysis of various composite cylindrical shells in the literature. Therefore, it is not possible to review all of them here, only some of them are given in this section. More detailed descriptions on the development of researches on this subject may be found in several monographs respectively by Leissa [1], Quta [2], Reddy [3], Carrera [4], and some review or survey articles [5–7]. As far as the shell deformation theories reported in previous studies are concerned, there are three major theories which are usually known as: the Classical Shell Theory (CST) [8,9], the First-order Shear Deformation Theory (FSDT) [10–12] and the Higher-order Shear Deformation Theory (HSDT) [3,5,13–16]. Researchers have found that application of classical thin shell theory to thick shells could lead to as much as 30% or more errors in natural frequencies. In addition, as pointed out by Qu et al. [17], these HSDTs are computationally more demanding than those FSDTs. Furthermore, from the existing literature, we can know that the first-order theory with proper shear correction factors is adequate for the prediction of the global behaviors of moderately thick shells. Therefore, in present work, the first-order shear deformation shell theory is just employed to formulate the theoretical model.

Apart from the aforementioned shear deformation theories, it has also been of great interest for researchers to develop an

* Corresponding author. Tel.: +86 451 82589199.
E-mail address: guoyongjin@hrbeu.edu.cn (G. Jin).

0263-8223/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2013.09.044

accurate and efficient method which can be used to determine the vibration behaviors of FG cylindrical shells. So far, many computational methods are available for the vibration analysis of cylindrical shells, such as the Rayleigh–Ritz method [8,9], Differential Quadrature (DQ) method [11,18] Galerkin method [19], Finite element method [20–22], wave propagation approach [23], meshless method [7,24,25], discrete singular convolution (DSC) method [26], general domain decomposition method [10], and various hybrid methods [22,25,27,28]. From the review of the literature, it appears that despite a variety of methods for analytical and computational analysis of cylindrical shell structures, it is still of need and great significance to develop a simple and efficient numerical method for vibration analysis of FGM shells. Therefore, the purpose of the present work is to introduce the Haar wavelet approach for the free vibration analysis of FGM cylindrical shells.

Wavelet analysis is a mathematical branch and has been developing rapidly. In the last several decades, wavelets methods have been developed as a new powerful tool for mathematical analysis and engineering computation. The current wavelets-based approaches include wavelets-collocation method [29–31], wavelet-finite elements method [32,33], etc. In most wavelet-based methods, the calculation of the wavelet connection coefficients is a complicated problem [34]. Obviously, attempts to simplify solutions based on the wavelet methods are required. Recently, the Haar wavelet which is originally introduced by Alfred Haar in 1910 has drawn considerable attention. One possible reason is that this kind of wavelet demonstrates its mathematical simplicity. The initial work in system analysis using Haar wavelets was done by Chen and Hsiao [35]. They recommended an operational matrix of integration based on the Haar wavelets. Later, Hsiao and Wang [36,37] developed this theory and proposed the Haar product matrix and coefficient matrix. Since the solution procedure is simple and direct, the Haar wavelet has been proven to be an effective tool for solving various problems, such as differential and integral equations [38,39], Poisson equations and biharmonic equations [40], eigenvalues of regular Sturm–Liouville problems [41] and high order differential equations [42], functionally graded plates [43], damage evaluation of plates [44], and non-uniform and functionally graded beams [45,46]. It is worth pointing out that Majak et al. [47] developed this method and introduced it for solving solid mechanics problems, and proposed the Haar wavelet discretization method for solving differential equations based on the weak formulation [48].

In this present work, a numerical discretization approach based on the Haar wavelet is introduced for the modeling and vibration analysis of FG cylindrical shells. The material properties of the shells are assumed to vary continuously in the thickness direction according to general four-parameter power-law distributions in terms of volume fractions of the constituents. The effects of shear deformation are considered. In order to test the convergence, efficiency and accuracy of the proposed method, some numerical examples are presented for the free vibrations of FG cylindrical shells. The effects of some geometrical and material parameters on the natural frequencies are also discussed. The main aim of this paper is to demonstrate a convenient and efficient application of the Haar wavelet discretization method to the free vibration analysis of FG cylindrical shells.

2. Theoretical formulations

2.1. The Haar wavelet series and their integrals

The Haar wavelet is one of the simplest orthonormal wavelet with a compact support. The Haar wavelet family is defined for $\xi \in [0,1]$ as follows [40,45,48]:

$$h_i(\xi) = \begin{cases} 1 & \xi \in [\xi^{(1)}, \xi^{(2)}] \\ -1 & \xi \in [\xi^{(2)}, \xi^{(3)}] \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

where the notations $\xi^{(1)} = \frac{k}{m}$, $\xi^{(2)} = \frac{k+0.5}{m}$, $\xi^{(3)} = \frac{k+1}{m}$ are introduced. The integer $m = 2^j$ ($j = 0, 1, \ldots, J$) is the factor of scale, where J is the maximal level of resolution. $k = 0, 1, \ldots, m-1$ is the translation parameter. The index i is calculated according to the formula $i = m + k + 1$, the maximal value is $i = 2M$, which is 2^{J+1}; the minimal value is $i = 2$ (then $m = 1$, $k = 0$). The case $i = 1$ corresponds to the scaling function:

$$h_1(\xi) = \begin{cases} 1 & 0 \leqslant \xi \leqslant 1 \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

The interval [0,1] is divided into $2M$ subintervals of equal length $\Delta x = 1/2M$; the collocation points are given as:

$$\xi_l = \frac{(l-0.5)}{2M}, \quad l = 1, 2, \ldots, 2M \tag{3}$$

The Haar coefficient matrix $\boldsymbol{H}$ is defined as $\boldsymbol{H}(i,l) = h_i(\xi l)$. For any square integrable function $y(x) \in L^2[0,1]$ in the interval [0,1], it can be expanded into the Haar wavelet series of infinite terms. If $y(x)$ is piecewise constant by itself, or may be approximated as piecewise constant during each subinterval, then $y(x)$ will be truncated with finite terms, that is

$$y(x) = \sum_{i=1}^{2M} a_i h_i(x) \tag{4}$$

where a_i are unknown wavelet coefficients. If we want to solve an n-th order PDE, the following integrals are required [42]

$$p_{\alpha,i}(x) = \underbrace{\int_0^{\xi}\int_0^{\xi}\cdots\int_0^{\xi}}_{\alpha-times} h_i(t)dt^{\alpha} = \frac{1}{(\alpha-1)!}\int_0^{\xi}(x-t)^{\alpha-1}h_i(t)dt$$
$$\alpha = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, 2M. \tag{5}$$

In Eq. (5), $p_{0,i}(x) = h_i(t)$. These integrals can be calculated analytically. In the case $i = 1$, we have $p_{\alpha,i}(\xi) = \xi^{\alpha}/\alpha!$; and in the case $i > 1$ we obtain the integrals as follows [45]:

$$p_{n,i}(\xi) = \begin{cases} 0 & \xi < \xi^{(1)} \\ \frac{1}{n!}(\xi-\xi^{(1)})^n & \xi^{(1)} < \xi < \xi^{(2)} \\ \frac{1}{n!}[(\xi-\xi^{(1)})^n - 2(\xi-\xi^{(2)})^n] & \xi^{(2)} < \xi < \xi^{(3)} \\ \frac{1}{n!}[(\xi-\xi^{(1)})^n - 2(\xi-\xi^{(2)})^n + (\xi-\xi^{(3)})^n] & \xi > \xi^{(3)} \end{cases} \tag{6}$$

For solving boundary value problems, the value $P_{a,i}(0)$ and $P_{a,i}(1)$ should be calculated in order to satisfy the boundary conditions. Substituting the collocation points in Eq. (3) into Eq. (6) yields

$$\boldsymbol{P}^{(\alpha)}(i,l) = p_{\alpha,i}(\xi) \tag{7}$$

where $\boldsymbol{P}^{(\alpha)}$ is a $2M \times 2M$ matrix. It should be noted that calculations of matrices $\boldsymbol{H}(i,l)$ and $\boldsymbol{P}^{(\alpha)}(i,l)$ must be carried out only once.

2.2. Description of the model

Let us consider a FG shell of revolution with uniform thickness h. A differential element of the shell is depicted in Fig. 1. The length and mean radius of the shell are represented by L and R, respectively. The reference surface of the shell is taken to be at its middle surface where an orthogonal coordinate system (x, θ and z) is fixed. The x-coordinate is taken in the axial direction of the shell, where the θ and z axes are respectively in the circumferential and radial directions.