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Delamination in laminated plates using the 4-noded quadrilateral QLRZ plate element based on the refined zigzag theory

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ABSTRACT

A numerical method based on the Refined Zigzag Theory (RZT) to model delamination in composite laminated plate/shell structures is presented. The originality of this method is the use of 4-noded quadrilateral plate finite elements whit only seven variables per node to discretize the plate/shell geometry. The ability to capture the relative displacement between consecutive layers in fracture mode II and III is the more important advantage of this element, denoted QLRZ [1].

A continuum isotropic damage model [2] is used to model the mechanical behavior of the plies. The material non-lineal problem is solved with the modified Newton–Raphson method.

The RZT plate theory, the QLRZ finite element and the isotropic damage model are described in this work. Also, the implicit integration algorithm is presented. The performance of the numerical model is analyzed by studying the delamination in a rectangular plate for two different laminates, using the 3D analysis as the reference solution.

1. Introduction

Delamination [3] is a dangerous failure mode in laminated composite materials and is normally characterized by a relative displacement between layers due to a loss of adherence. Local forces, thermal actions and low energy impacts may serve as sources of delamination during the transportation, storage or service life of the structural member. In addition, geometry discontinuities such as access holes, notches, free edges or bonded and bolted joints can also induce delamination due to high stress gradients. Once delamination has occurred, the initial stiffness of the structure could be considerably reduced which can induce the structural failure by other phenomena as buckling, excessive vibration or fatigue.

During the design phases of laminated structures, it is important to know how the global response of the structure will be affected by delamination. Thus, much effort and time is been invested to develop numerical tools that can predict delamination in an effective and efficient manner.

The more common procedures to model delamination are based on the fracture mechanics or the damage mechanics. The virtual crack closure technique (VCCT) [4–6] and the cohesive finite elements [7–11] are some typical examples. Each of these techniques has their own drawbacks, but they share one of the most inefficient features: the need to place interface fracture or cohesive finite elements between the plies where delamination is expected to occur. Because of the delamination path is normally unknown, it is necessary to place interface elements between all layers, which leads to an increase of computational resources needed to carry out the simulation, specially in laminates with many plies. In order to avoid the above-mentioned disadvantage, Martinez et al. [12] have studied delamination under the continuum mechanics using a 3D finite element method and an isotropic damage model to manage material degradation.

The capabilities of 3D models are well known. However, the computational resources needed for modeling non-linear problems grow significantly when 3D finite elements are used to discretize the structure. Although there are several cases where a 3D analysis is indispensable, for instance for studying the delamination in bonded joints, it is almost computationally impossible to use them for large laminated composite structures with tens of layers such as wind turbine blades or aircraft fuselage. For these kinds of structures, more simplified models should be used.

Some examples of simple models used to simulate laminated composite plate/shell structures are the First Order Shear Deformation Theory (FSDT) [13,14], the Layer-Wise theories (LWT) [15–18], the ZigZag (ZZ) theories [17,19–21] and the *Refined ZigZag Theory* (RZT) [22–26].

Despite the simplicity of the FSDT theory, it is well documented [1,27] that this model gives wrong predictions for highly heterogeneous laminates. In addition, the FSDT is unable to capture delamination because of its linear kinematics assumptions.

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LWT models describe separately the displacement field within each ply, which leads to a high level of refinement of the kinematics. Because of that, they can reproduce with high precision the complex kinematics of highly heterogeneous laminates, and also simulate the delamination phenomenon [28,29]. However, since the number of unknowns is proportional to the number of analysis sublayers (that may be not coincident with the number of physical layers), the computational cost increases with the number of subdivisions.

ZZ theories are an attractive compromise between the high accuracy of LWT and the computational efficiency of FSDT. The kinematics is defined as a superposition of a piecewise linear displacement functions over a linear, quadratic or cubic displacement field along the thickness direction. The number of kinematics variables is independent of the number of layers, which favors the efficiency. Despite its good performance, they present some difficulties to model correctly some boundary condition. So far, the use of the ZZ theories to model delamination in beams and plates has been quite limited. A ZZ model to simulate delamination has been developed by Icardi and Zardo [30].

The kinematics proposed by the RZT theory is defined by a superposition of a linear piecewise zigzag function over the FSDT displacement fields. Since RZT is an improvement of the ZZ theories, the number of variables is also independent of the number of plies. However, unlike the ZZ, all boundary conditions, including the fully clamped condition, can be simulated effectively as it was demonstrated in the original paper [23,24]. Oñate et al. [25,27] and Eijo et al. [1] have taken the RZT as the basis for developing linear beam (LRZ) and quadrilateral plate (QLRZ) finite elements, respectively.

Eijo et al. [31] have extended the LRZ element to simulate delamination in laminated beams. Since the vertical displacement is defined constant along the thickness and the transversal in-plane displacement is not considered for the RZT beam theory, the proposed methodology is limited to model only the fracture mode II. In addition, delamination in highly heterogeneous laminates, i.e. laminates where the shear modulus of the laminae differ from each other in many orders of magnitude can not be correctly simulated employing this technique. However, that is not the case of composite laminates where the shear modulus of laminae does not differ generally in more than one order of magnitude [32]. For this model, delamination can happen at any place within the laminate, thus, it is not necessary to predefine the path where crack is expected to occur. An isotropic damage model was used to manage the non-linear material behavior. It was demonstrated that, in order to be able for capturing relative displacements between layers, the piecewise zigzag function must be updated in terms of the damage level of the material. In [31] it was shown not only the ability to capture the relative displacement between layers, but also the efficiency of the numerical model based on the RZT theory.

In this paper, we present the extension of the beam delamination model of [31] to plate/shell structures using the QLRZ element. Unlike the beam theory, the transversal in-plane displacement is taken into account for the plate theory, which allows simulating not only the fracture mode II but also mode III. For the same reason as in beams, it is not possible to predict the opening fracture mode. The non-linear material behavior is modeled using an isotropic damage model. The non-linear problem is solved by the modified Newton–Raphson method. The paper describes the RZT plate theory, the formulation of the QLRZ finite elements and the isotropic damage model. Also, the implicit integration algorithm is described. Finally, the performance of the proposed numerical model is shown by modeling delamination in a simply supported rectangular plate with a center hole for two different laminates. The reference solution is a 3D analysis using eight-noded hexahedral elements.

2. Refined Zigzag Theory (RZT) for plate and QLRZ plate/shell element

2.1. RZT plate kinematics

A laminated plate formed by N analysis layers of thickness h^k is considered. The number of analysis layer may be not coincident with the number of physical layers. The reference coordinate system is the 3D Cartesian system (x,y,z), where x–y are set as the in-plane coordinates and z is the thickness coordinate.

The plate displacement field proposed by the RZT is defined as (Fig. 1)

$$\begin{aligned} u^k(x,y,z) &= u_0(x,y) - z\cdot\theta_x(x,y) + \bar{u}^k(x,y,z) \\ v^k(x,y,z) &= v_0(x,y) - z\cdot\theta_y(x,y) + \bar{v}^k(x,y,z) \\ w(x,y) &= w_0(x,y) \end{aligned} \tag{1a}$$

where the linear piecewise zigzag functions are

$$\begin{aligned} \bar{u}^k &= \phi_x^k(z)\cdot\psi_x(x,y); \quad k = 1, N \\ \bar{v}^k &= \phi_y^k(z)\cdot\psi_y(x,y) \end{aligned} \tag{1b}$$

and superscript k indicates quantities within the kth layer with $z_k \leqslant z \leqslant z_{k+1}$, and z_k is the vertical coordinate of the kth interface. The *uniform axial displacements* along the coordinate directions x and y are u_0 and v_0, respectively; θ_x and θ_y represent the *average bending rotation* of the transverse normal about the negative y and positive x directions; and w_0 is the *transverse deflection*.

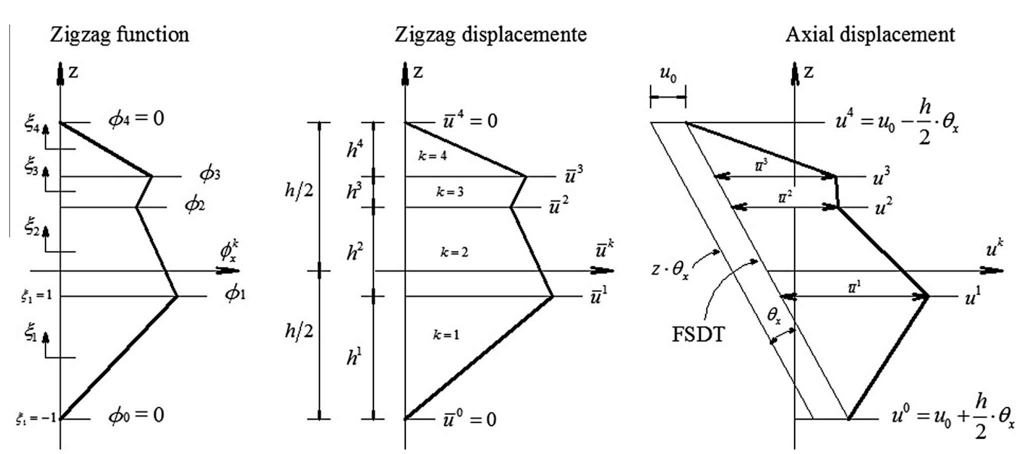

Fig. 1. RZT kinematics.