Composite Structures 108 (2014) 472–492

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Thermal buckling of functionally graded plates using a local Kriging meshless method

L.W. Zhang [a], P. Zhu [b], K.M. Liew [c,*]

[a] College of Information Technology, Shanghai Ocean University, 999 Huchenghuan Road, Shanghai 201306, China
[b] College of Civil Engineering, Hunan University, 1 South Lushan Road, China
[c] Department of Civil and Architectural Engineering, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong

ARTICLE INFO

Article history:
Available online 27 September 2013

Keywords:
Functionally graded plates
Buckling analysis
Kriging interpolation
Local Kriging meshless method
First-order shear deformation plate theory

ABSTRACT

The mechanical and thermal buckling behaviors of ceramic–metal functionally grade plates (FGPs) were studied by using a local Kriging meshless method. The local meshless method was developed based on the local Petrov–Galerkin weak-form formulation combined with shape functions having the Kronecker delta function property, constructed by the Kriging interpolation. The cubic spline function of high continuity was used as the weight function to simplify the local weak form of governing equations with the integration on the internal boundaries vanishing. The transverse shear strains of FGPs were incorporated by employing the first-order shear deformation plate theory and plate material properties were assumed to change exponentially along the thickness direction. Convergence and comparison studies examined the stability and accuracy of the presented method. Two types of FGMs, Al/Al_2O_3 and Ti–6Al–4V/Aluminum oxide, were chosen for mechanical and thermal buckling analyses. The influences of volume fraction exponent, boundary condition, length-to-thickness ratio and loading type on the buckling behaviors of FGPs were discussed.

1. Introduction

Functionally graded materials (FGMs), composed of metal and ceramic typically, have been widely used in many engineering applications and working environments, especially in high temperature and large temperature gradient environments. The ceramic provides high temperature–resistant and high corrosion–resistant while metal gives high roughness. Since the material properties of functionally graded plates (FGPs) normally vary smoothly and continuously along the thickness direction according to a power-law distribution of the volume fraction of ceramic, FGPs is superior to conventional fiber-based laminated composites by avoiding delamination-related problems.

Due to advantages of FGMs, a number of investigations dealing with thermal behaviors of functionally graded plates and shells were published in scientific literature, such as static and transient thermo-elastic responses [1], nonlinear free flexural vibrations in thermal environments [2], and thermal postbuckling behaviors with temperature-independent material properties [3,4] and with temperature-dependent properties [5–7]. For buckling analysis of FGPs, research has been conducted with different geometries, loading types, and plate theories by analytical and numerical methods. Birman [8] studied the buckling behavior of functionally graded hybrid composite plates with two classes of fibers, subjected to uniaxial compression. Feldman and Aboudi [9] performed the elastic buckling analysis of FGPs under in-plane compressive loadings with a method based on a combination of micromechanical and structural approaches. Javaheri and Eslami [10,11] carried out the mechanical and thermal buckling analysis of rectangular FGPs based on the classical plate theory. Additionally, they investigated buckling of FGPs under several thermal loadings based on the higher order shear deformation plate theory [12]. Lanhe [13] studied the thermal buckling of moderately thick rectangular FGPs with various types of thermal loadings, relative thicknesses, gradient indexes and plate aspect ratios based on the first order shear deformation plate theory. Buckling of circular FGPs under uniform radial compression was studied by Najafizadeh and Eslami [14]. Using the third-order shear deformation theory, Ma and Wang [15] and Saidi et al. [16] analyzed the buckling characteristics of circular FGPs. Chen and Liew [17] investigated the buckling of rectangular FGPs subjected to nonlinear distributed in-plane edge loads with a meshfree method based on the radial basis function. Na and Kim [18] conducted the three-dimensional thermal buckling analysis of FGPs using the finite element method. Ganapathi and Prakash [19] analyzed the thermal buckling of skew FGPs in terms of the first-order shear deformation theory in conjunction with the finite element approach. Recently, buckling of circular and square FGPs under uniaxial thermal and

* Corresponding author. Tel.: +852 34426581.
E-mail address: kmliew@cityu.edu.hk (K.M. Liew).

0263-8223/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2013.09.043

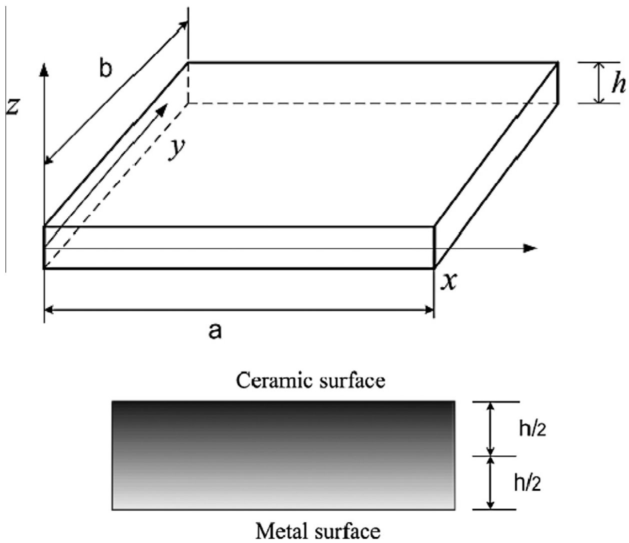

Fig. 1. Details of a functionally graded plate.

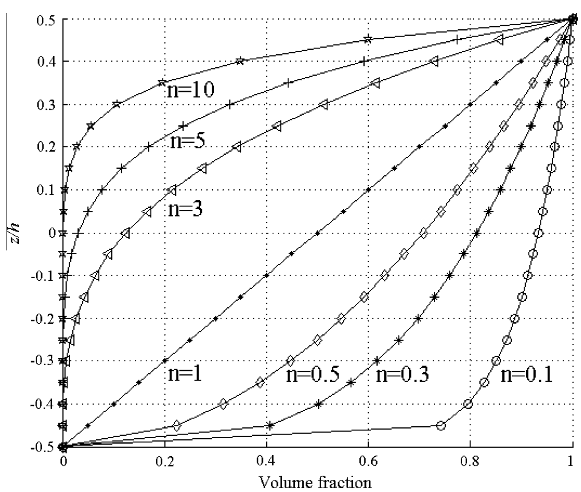

Fig. 2. Variation of the volume fraction V_c through the thickness of a functionally graded plate.

mechanical loads was investigated by Zhao et al. [20] by using the element-free *kp*-Ritz method based on the first-order shear deformation plate theory. With the element-free *kp*-Ritz method, Zhao and Liew have also performed thermal buckling and postbuckling analyses for functionally graded shells [21–23]. In a recent article [24], a review of element-free methods and their applications in the buckling analysis for functionally graded plates was provided.

Meshless methods, representing a problem domain by a set of scattered nodes instead of using meshes for discretization in finite element analysis, have been continuously proposed and applied to scientific and engineering fields. Shape functions used in meshless methods are totally in terms of scattered nodes among which no connectivity exists. Thus, those drawbacks of mesh-based methods such as distortion of meshes causing inaccurate and unstable solutions [25] and high costs in creating remeshing [26] can be avoided. In the last decade, Kriging interpolation developed by Krige [27] who presented the use of moving averages to avoid systematic errors in interpolation was applied in the framework of meshless methods for constructing shape functions. Kriging interpolation is a much effective technique to construct simple, efficient and stable shape functions having the delta function property which benefits enforcement of essential boundary conditions. In addition, the established Kriging-based shape functions have properties of the partition of unity and reproducibility which make any functions included in the basis be exactly reproduced. With these beneficial features, Kriging-based shape functions are widely used in different types of meshless formulations for solving partial differential equation problems. Gu [28] first applied the moving Kriging interpolation to construction of shape functions which were then introduced to the global Galerkin mesheless formulation to solve steady-state heat conduction problems. Tongsuk and Kanok-Nukulchai [29,30] further applied the Kriging-based element-free Galerkin (EFG) method to one- and two-dimensional elasticity problems and performed the parametric studies with this method. Sayakoummane and Kanok-Nukulchai [31] used the Kriging-based EFG method to conduct bending analysis of degenerated shell structures. As well, plate structures were recently analyzed in conjunction with the moving Kriging-based EFG methods by Bui et al. [32–34]. Moreover, the moving Kriging interpolation was combined with the boundary integral equation (BIE) method to construct a boundary-type meshfree method for two-dimensional potential problems [35]. Kriging interpolation with an error-reproduction kernel method was applied to solving linear and nonlinear boundary value problems [36]. A method based on Kriging interpolation and finite cover technique was developed for fracture analysis [37]. Instead of global formulations described previously, Lam et al. [38] proposed a local weak-form meshless formulation incorporating Kriging-based shape functions to form a novel local Kriging (LoKriging) meshless method for two-dimensional structural analysis. The LoKriging method was further applied to static and dynamic problems of microelectromechanical systems device subjected to dynamic nonlinear loadings [25,39]. Recently, The Lokriging method was employed to solve two-dimensional and three-dimensional transient heat conduction problems [40], elastodynamic analysis for two-dimensional solids [41] and free vibration analysis of moderately thick functionally graded plates [42].

In this paper, the local Kriging meshless method is extended to investigation of buckling analysis of functionally graded plates under thermal and mechanical loadings based on the first-order shear deformation plate theory which incorporates transverse shear strains of FGPs. The shape functions in terms of a set of arbitrary distributed nodes are constructed by the Kriging interpolation

Table 1
Coefficients of Ti–6Al–4V and aluminum oxide.

Coefficients	Aluminum oxide				Ti–6Al–4V			
	E (N/m^2)	ν	κ (W/mK)	α (/K)	E (N/m^2)	ν	κ (W/mK)	α (/K)
P_{-1}	0	0	−1123.6	0	0	0	0	0
P_0	349.55×10^9	0.26	−14.087	6.8269×10^{-6}	122.56×10^9	0.2884	1.0	7.5788×10^{-6}
P_1	-3.853×10^{-4}	0	-6.227×10^{-3}	1.838×10^{-4}	-4.586×10^{-4}	1.121×10^{-4}	1.704×10^{-2}	6.638×10^{-4}
P_2	4.027×10^{-7}	0	0	0	0	0	0	-3.147×10^{-6}
P_3	-1.673×10^{-10}	0	0	0	0	0	0	0
P (300 K)	320.24×10^9	0.26	64.989	7.203×10^{-6}	105.7×10^9	0.298	6.112	6.941×10^{-6}