Composite Structures 108 (2014) 504–513

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

An exact finite strip for the calculation of initial post-buckling stiffness of shear-deformable composite laminated plates

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ARTICLE INFO

Article history:
Available online 1 October 2013

Keywords:
Exact strip
Shear-deformable plates
Symmetrically cross-ply laminates
Von-Karman's equations
Initial post-buckling analysis

ABSTRACT

The first order shear deformation theory (FSDT) is used to present the buckling and initial post-buckling characteristics of symmetrically cross-ply laminated plates. In the buckling phase the Von-Karman's equilibrium equation is solved exactly for the FSDT to obtain out-of-plane mode shapes and critical loads. The current post-buckling study is effectively a single-mode analysis, which is attempted by utilizing the so-called semi-energy method. The Von-Karman's compatibility equation is solved exactly in the post-buckling phase with the assumption that the deflected form immediately after buckling is the same as that obtained for buckling. The Principle of Minimum Potential Energy is invoked to solve for the unknown coefficients in the assumed out-of-plane deflections and rotation functions.

1. Introduction

Prismatic plates and plate structures are often employed in situations where they are subjected to in-plane compressive loading. In aerospace, in particular, the quest for efficient, light weight structures often leads to allowing for the possibility of local buckling and post-local-buckling at load levels lying between the design limit load and ultimate conditions. Thus it is important to accurately predict the buckling and post-buckling behavior of such structures.

The first person who developed the basic concepts of finite strip method (FSM) is Cheung [1]. Cheung's theory is based on the idea that the finite strip can be assumed as a kind of finite element in which special long elements called strips are used.

After Cheung, different variations/extensions of FSM are achieved by many researchers. Lau and Hancock [2], Wang and Dawe [3] and Zou and Lam [4] used FSM implementing Classical Plate Theory (CLPT), First-order Shear Deformation Theory (FSDT) and Higher-order Shear Deformation Theory (HSDT). The post-local-buckling behavior of elastic plates or plate structures is a geometric non-linear problem. The non-linearity occurs as a result of relatively large out-of-plane deflection, which necessitates the inclusion of non-linear terms in the strain–displacement equations. Many works have been done concerning the geometrically non-linearity response of the structures by using FSM. Early works are those of Graves-Smith and Sridharan [5,6] and Hancock [7]. These authors have assumed a plate with simply-supported ends subjected to progressive end shortening. Subsequently the post-buckling behavior of the structure is predicted by using FSM in which the in-plane displacement fields are postulated in addition to the out-of-plane displacement field. The lengthwise variations in the displacement fields are trigonometric functions, and the crosswise variations in both in-plane and out-of-plane displacement fields are simple polynomial functions. An energy-based method, referenced to as the semi-energy method by Rhodes and Harvey [8], was first used by Marguerre [9] and has since been used by various researchers. Rhodes' [10] and Chou and Rhodes' [11] papers are mostly based on the semi-energy method while containing some useful experimental data. Khong and Rhodes [12] have setup a computationally efficient approach to the post-buckling analysis of prismatic structural members. In this approach, a linear finite strip method, developed for the buckling analysis, based on the Principle of Minimum Potential Energy is employed to find the buckling eigenvector. This eigenvector is then used as the post-buckled deflected shape in a single-term post-buckling analysis based on the Principle of Minimum Potential Energy. The analysis is simplified by the assumption that stresses in the direction perpendicular to loading and shear stresses have negligible effects. This approach can be considered as a lower bound method of post-buckling analysis (i.e. the post-buckling stiffness of the structure is underestimated by this approach). The method is applied to plain and stiffened channel sections as well as Z-sections.

Ovesy et al. [13–15] have developed a Semi-energy post-local-buckling FSM (S-e FSM) in which the out-of-plane displacement of the finite strip is the only displacement which is postulated by a deflected form as distinct to that mentioned so far in the paper

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http://dx.doi.org/10.1016/j.compstruct.2013.09.049

with respect to the previously developed methods. The developed semi-energy FSM (S-e FSM) has been applied to analyze the post-local-buckling behavior of thin flat plates [5], open channel section [14] and box section struts [15]. Ovesy and Ghannadpour [16–18] have also developed a single-term Full-analytical FSM (F-a FSM) based on CLPT in which the Von-Karman's equilibrium equation is solved exactly and so the buckling mode shapes and loads are obtained with very high accuracy. The obtained mode shapes are then used in the post-buckling phase and the Von-Karman's compatibility equation is solved exactly and the in-plane displacements are obtained with very high accuracy.

Ghannadpour and Ovesy [19,20] have further enhanced their post-buckling exact CLPT finite strip method by taking into account a combination of the first, second and higher (if required) modes of buckling. The Von-Karman's compatibility equation is then solved by substituting the combined deflection function, and thus the post-buckling behavior of isotropic structures are investigated with high accuracy. Ghannadpour and Ovesy [21] have upgraded their exact strip for calculating the relative post-buckling stiffness of composite plates based on the CLPT. Ovesy et al. [22] have developed a new exact finite strip for investigating the buckling behavior of moderately thick composite plates and plate structures based on the FSDT. In this method the Von-Karman's equilibrium set of equations has been solved analytically and the buckling mode shapes has been found with very high accuracy. Ghannadpour et al. [23] have recently extended their exact method for non-linear initial post-buckling behavior of isotropic plates based on the FSDT.

In this paper, an exact finite strip is modeled using the FSDT. In this model for the first time, the buckling and initial post-buckling analysis of moderately thick composite plates have been presented. In the buckling phase, with the assumption that the two loaded ends are simply supported and the other two ends have arbitrary out-of-plane boundary conditions, the Von-Karman's equilibrium set of equations has been solved exactly. A transcendental stiffness matrix is obtained for an individual composite strip. The analytical solution function of the set of equations has been developed to a more general function which contains all of the solution conditions. Thus the general out-of-plane buckling modes are obtained with very high accuracy. The Von-Karman's compatibility equation is then solved exactly to obtain the general form of in-plane displacement fields in the initial post-buckling region.

2. Theoretical developments of the F-a FSM

In this section, the fundamental elements of the theory for the developed exact finite strip in buckling and initial post-buckling problems are outlined. It must be noted that a perfectly flat exact strip made up of orthotropic layers constructing symmetric laminates, which possesses out-of-plane orthotropy, is assumed throughout the theoretical developments of this paper. The so-called exact finite strip is assumed to be simply supported out-of-plane at the loaded ends and arbitrary out-of-plane boundary condition at the other two edges. It is important to mention that the plate is assumed to be moderately thick, thus the FSDT is applied in the remaining of the paper.

2.1. Basic formulation of the problem

An exact finite strip, as schematically shown in Fig. 1, and is of length L, width b and thickness h is assumed. As mentioned earlier, the finite strip is simply supported out-of-plane at both ends, i.e.

$$w = \varphi_y = M_x = 0 \tag{1}$$

It must be noticed that the FSDT is applied, thus

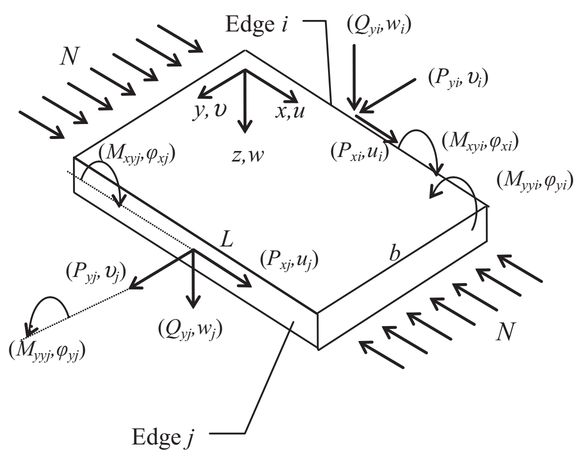

Fig. 1. A typical exact finite strip.

$$\hat{u} = u + z\varphi_x, \quad \hat{\upsilon} = \upsilon + z\varphi_y, \quad \hat{w} = w \tag{2}$$

where $\hat{u}, \hat{\upsilon}$ and $\hat{w}$ are components of displacement at a general point, whilst u, υ and w are similar components at the middle surface ($z = 0$), φ_x is the rotation of a transverse normal about the axis y and φ_y is the rotation of a transverse normal about the axis x. In the FSDT it is assumed that the whole transverse shear components cannot be neglected, thus, with respect to this assumption, the stress–strain relationship at a general point for a symmetrically cross-ply laminated composite plate becomes:

$$\bar{\boldsymbol{\sigma}} = \begin{Bmatrix} \bar{\sigma}_{xx} \\ \bar{\sigma}_{yy} \\ \bar{\tau}_{yz} \\ \bar{\tau}_{xz} \\ \bar{\tau}_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & 0 & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \overline{Q}_{44} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{55} & 0 \\ 0 & 0 & 0 & 0 & \overline{Q}_{66} \end{bmatrix} \cdot \bar{\boldsymbol{\varepsilon}}; \quad \bar{\boldsymbol{\varepsilon}} = \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\gamma}_{yz} \\ \bar{\gamma}_{xz} \\ \bar{\gamma}_{xy} \end{Bmatrix} \tag{3}$$

where $\bar{\boldsymbol{\sigma}}$ and $\bar{\boldsymbol{\varepsilon}}$ are the stresses and strains at a general point and $\overline{Q}_{ij}(i,j = 1,2,4,5,6)$ are plate stiffness coefficients.

Internal forces and moment acting on the edges of a strip are expressed in terms of forces and the moments per unit distance along the strip edge. The forces and moments intensities are related to the internal stress by the equations

$$\langle N_{xx}\ N_{yy}\ N_{xy}\rangle^T = \int_{-h/2}^{h/2} \langle \bar{\sigma}_{xx}\ \bar{\sigma}_{yy}\ \bar{\tau}_{xy}\rangle^T \mathrm{d}z \tag{4a}$$

$$\langle M_{xx}\ M_{yy}\ M_{xy}\rangle^T = \int_{-h/2}^{h/2} \langle \bar{\sigma}_{xx}\ \bar{\sigma}_{yy}\ \bar{\tau}_{xy}\rangle^T z\mathrm{d}z \tag{4b}$$

$$\langle Q_x\ Q_y\rangle^T = K\int_{-h/2}^{h/2} \langle \bar{\tau}_{xz}\ \bar{\tau}_{yz}\rangle^T \mathrm{d}z \tag{4c}$$

where K is a shear correction factor and can be obtained from the method developed in [24].

It is noted that the stress and strain vectors $\bar{\boldsymbol{\sigma}}' = \langle \bar{\sigma}_{xx}\ \bar{\sigma}_{yy}\ \bar{\tau}_{xy}\rangle^T$ and $\bar{\boldsymbol{\varepsilon}}' = \langle \bar{\varepsilon}_{xx}\ \bar{\varepsilon}_{yy}\ \bar{\gamma}_{xy}\rangle^T$ include the components corresponding to the membrane and bending contributions as outlined below

$$\bar{\boldsymbol{\sigma}}' = \boldsymbol{\sigma} + \boldsymbol{\sigma}_b; \quad \bar{\boldsymbol{\varepsilon}}' = \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}_b \tag{5}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ correspond to the membrane contribution, and $\boldsymbol{\sigma}_b$ and $\boldsymbol{\varepsilon}_b$ relate to the bending and twisting actions. It is noted that the membrane strain $\boldsymbol{\varepsilon}$ can be subdivided into its linear $\boldsymbol{\varepsilon}_l$ and non-linear $\boldsymbol{\varepsilon}_{nl}$ component as given below

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_l + \boldsymbol{\varepsilon}_{nl} = \begin{Bmatrix} u_{,x} \\ \upsilon_{,y} \\ u_{,y} + \upsilon_{,x} \end{Bmatrix} + \begin{Bmatrix} \frac{1}{2}w_{,x}^2 \\ \frac{1}{2}w_{,y}^2 \\ w_{,y}w_{,x} \end{Bmatrix} \tag{6}$$