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Nonlinear forced vibration response of bimodular laminated composite plates

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ABSTRACT

The nonlinear forced vibration characteristics of bimodular material laminated cross-ply composite plates subjected to periodic excitation are investigated. The analysis is carried out using Bert's constitutive model employing first order shear deformation theory based finite element method (FEM) including von Kármán geometric nonlinearity. The periodic response is obtained using shooting technique coupled with Newmark time marching, arc length and pseudo-arc length continuation algorithms. The second order differential equation of motion is solved directly without transforming to first order differential equations thereby preserving the banded nature of equations. The nonlinear periodic vibration characteristics such as steady state response history, phase plane plots and frequency spectra are presented. A detailed parametric study is carried out to analyse the influence of bimodularity, aspect ratio, thickness ratio, excitation amplitude, support conditions and lamination scheme on the forced vibration response. The significant difference in the positive/negative half cycle amplitudes due to bimodularity is predicted with/without geometric nonlinearity. The through the thickness variation of fiber direction normal strain is presented to show the extent of assignment of tensile/compressive properties and restoring force due to geometric nonlinearity. Further, the unstable regions of the frequency response curves between turning points could not be traced due to the perturbations stemming from the numerical truncation in finite element discretization/solution of equations and the bimodular action resulting in deviation of solution in shooting iterations leading to convergence failure.

1. Introduction

Many fiber-reinforced composites such as Glass/Epoxy, Carbon/Carbon, Graphite/Epoxy, and Human Cortical bone (femur) exhibit different stress–strain relationship in tension and compression and are referred to as bimodular materials [1–3]. Laminated composite plates made of such materials are extensively used in automobiles, aviation, missiles, rocket nozzles and biomechanical systems, etc. There are two basic material models describing the behavior of bimodular materials: one proposed by Ambartsumyan and Khachatryan [4] and the second one by Bert [5]. Ambartsumyan and Khachatryan model [4] is based on the signs of principal stresses and is mainly applicable to isotropic materials. Bert's constitutive model [5] is based on the sign of fiber direction normal strain and is applicable to orthotropic materials. The results from Bert's model have been found to agree well with experimental results [5] and it is being extensively used for research on laminated bimodular composites. The analysis of structures made of bimodular laminates are complex as compared to unimodular laminates since the elastic moduli depends upon the sign of fiber direction strain which is unknown *a priori*.

A lot of research has been done on the static bending analyses of bimodular laminates based on either the classical plate theory or first order shear deformation theory [6–9]. The higher-order theories for the determination of displacements and neutral surface locations of cross ply bimodular laminates have also been used by researchers [10–13]. The hybrid stress plate bending element is employed by Tseng and Ziang [14] for stress analysis of bimodular laminated rectangular plate whereas failure and damage analysis is carried out using layerwise theory by Zinno and Greco [15]. The static analysis of bimodular laminates including geometric nonlinearity has received the attention of few researchers [16–19].

The free flexural vibration analysis of single-layer orthotropic/two layered cross ply bimodular rectangular plates [20,21] and panels [22] has been carried out using first-order shear deformation theory using analytical/finite element methods. Reddy [23] carried out transient response analysis of single layer orthotropic and two layer cross-ply bimodular laminated rectangular plates based on first order shear deformation theory. Doong and Chen [24] analysed axisymmetric free vibration behavior of single layer orthotropic bimodular circular plate using first order shear deformation theory whereas the asymmetric free vibration and dynamic

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stability behavior have been studied by Chen and Chen [25]. Doong and Fung [26] studied the vibration and buckling of single-layer orthotropic/two layered cross ply bimodular rectangular plates using higher order shear deformation theories. Patel et al. [27] carried out free flexural vibration analysis of bimodular laminated angle ply plates based on higher order shear deformation theory. It is observed in their analysis that time period and amplitude are different for positive and negative half cycles for cross-ply rectangular plates but are same for cross-ply square and angle ply plates. Flexural vibration behavior of bimodular laminated composite cylindrical panels and the effect of bimodularity on the steady state response have been analysed by Khan et al. [28]. An approach based on Galerkin method in time domain has been employed for the frequency response of bimodular material laminated cylindrical panels by Khan et al. [29]. They further extended their work and analysed the effect of bimodularity on frequency response of cross-ply conical panels [30]. It is observed for bimodular laminates that the frequency response curves of positive and negative halve cycles are significantly different.

It is concluded from the literature review that the dynamic behavior of bimodular composite laminates considering geometric nonlinearity has not been investigated. For vibrations with amplitude of the order of plate thickness, consideration of geometric nonlinearity becomes important especially for thin plates. The geometrically nonlinear dynamic behaviour of plates is encountered in many applications wherein excitation amplitudes are greater and linear theories fail in predicting deflections, strains, stresses and frequencies to the desired level of accuracy.

The steady state frequency response analysis of bimodular structures is a challenging task due to non smooth nonlinear nature of governing equations. In this paper, the combined influence of bimodularity and geometric nonlinearity on the dynamic characteristics of laminated plates subjected to harmonic excitation is investigated. The study is carried out using first-order shear deformation based finite element and Bert's constitutive model. The governing equations in time-domain are solved using Newmark's time integration coupled with Newton Raphson method. The periodic responses are obtained using shooting technique. A parametric study is carried out to investigate the effects of bimodularity, aspect ratio, thickness ratio, lamination scheme, boundary conditions and material on the frequency response, steady state response history, phase plane plots, presence of higher harmonics and fiber direction normal strain distribution. The frequency response is compared with that obtained without considering geometric nonlinearity.

2. Formulation

The displacement field of a bimodular material laminated composite plate (Fig. 1) is taken as:

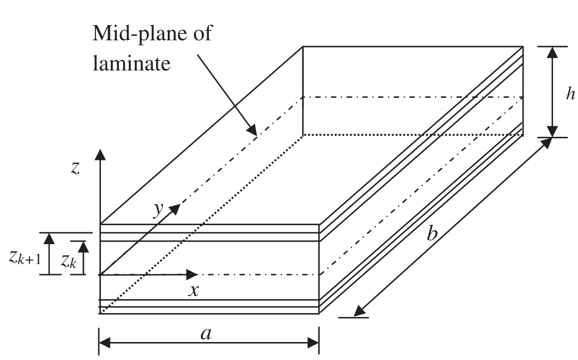

Fig. 1. Geometry and coordinate system of bimodular composite plate.

$$\begin{aligned} u\,(x,y,z,t) &= u_0\,(x,y,t) + z\theta_x\,(x,y,t) \\ v\,(x,y,z,t) &= v_0\,(x,y,t) + z\theta_y\,(x,y,t) \\ w\,(x,y,z,t) &= w_0\,(x,y,t) \end{aligned} \tag{1}$$

Here u_0, v_0, w_0 are the displacements of a generic point on the reference surface; θ_x and θ_y are the rotations of the normal to the reference surface about the y and x axes, respectively.

Using von Kármán's assumption for small strains and moderately large deflection, strain field in terms of mid-plane deformation can be written as

$$\{\varepsilon\} = \begin{Bmatrix} \boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{L} \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} z\boldsymbol{\varepsilon}_\mathbf{b} \\ \boldsymbol{\varepsilon}_\mathbf{s} \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{NL} \\ \mathbf{0} \end{Bmatrix} \tag{2}$$

where the in-plane $\boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{L}$, bending $\boldsymbol{\varepsilon}_\mathbf{b}$, transverse shear $\boldsymbol{\varepsilon}_\mathbf{s}$ and the nonlinear in-plane $\boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{NL}$ strain vectors are defined as

$$\boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{L} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}_\mathbf{b} = \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \end{Bmatrix},$$
$$\boldsymbol{\varepsilon}_\mathbf{s} = \begin{Bmatrix} \theta_x + \frac{\partial w_0}{\partial x} \\ \theta_y + \frac{\partial w_0}{\partial y} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{NL} = \begin{Bmatrix} \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix} \tag{3}$$

The fiber-governed constitutive relations for an arbitrary layer 'k' in the laminate (x,y,z) coordinate system can be expressed as

$$\{\boldsymbol{\sigma}\} = \{\,\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \tau_{xy} \quad \tau_{xz} \quad \tau_{yz}\,\}^T = [\overline{\mathbf{Q}}_{lk}]\{\boldsymbol{\varepsilon}\} \tag{4}$$

In Eq. (4), the terms of constitutive matrix $[\overline{\mathbf{Q}}_{\mathbf{lk}}]$ of the kth ply are referred to the laminate axes. The matrix $[\overline{\mathbf{Q}}_{\mathbf{lk}}]$ can be expressed in terms of the Young's moduli, shear moduli and Poison's ratios of the material in tension or compression depending upon the sign of fiber direction strain. The subscript l refers to the bimodular characteristics: $l = 1$ denotes the properties associated with fiber direction tension, $l = 2$ denotes the properties associated with fiber direction compression.

The kinetic energy of the plate is given by

$$T(\boldsymbol{\delta}) = \frac{1}{2}\int\int\left[\sum_{k=1}^{n}\int_{h_k}^{h_{k+1}} \rho_k\{\,\dot{u}_k \quad \dot{v}_k \quad \dot{w}_k\}\{\,\dot{u}_k \quad \dot{v}_k \quad \dot{w}_k\}^T dz\right]dxdy \tag{5}$$

where ρ_k is the mass density of the kth layer and h_k, h_{k+1} are the z-coordinates of the laminate corresponding to the bottom and top surfaces of the kth layer.

The total potential energy functional $U(\boldsymbol{\delta})$ consisting of strain energy and potential of transverse load is given by

$$U(\boldsymbol{\delta}) = \frac{1}{2}\int\int\left[\sum_{k=1}^{n}\int_{h_k}^{h_{k+1}}\{\sigma\}^T\{\varepsilon\}dz\right]dxdy - \int_A q w_0 dA \tag{6}$$

The total potential energy functional given by Eq. (6), using the approach of Rajasekaran and Murray [31], can be expressed as

$$\begin{aligned} U(\boldsymbol{\delta}) = {} & \{\boldsymbol{\delta}\}^T[(1/2)\mathbf{K} + (1/6)\mathbf{K}_1(\boldsymbol{\delta}) + (1/12)\mathbf{K}_2(\boldsymbol{\delta})]\{\boldsymbol{\delta}\} \\ & - \{\boldsymbol{\delta}\}^T\{\mathbf{F}\} \end{aligned} \tag{7}$$

where $[\mathbf{K}]$ is the linear stiffness matrix, $[\mathbf{K}_1]$ and $[\mathbf{K}_2]$ are nonlinear stiffness matrices linearly and quadratically dependent on the field variables, respectively, in positive/negative half cycle.

The governing equations of motion, considering dissipative forces, can be written as:

$$[\mathbf{M}]\{\ddot{\boldsymbol{\delta}}\} + [\mathbf{C}]\{\dot{\boldsymbol{\delta}}\} + [\mathbf{K} + (1/2)\mathbf{K}_1(\boldsymbol{\delta}) + (1/3)\mathbf{K}_2(\boldsymbol{\delta})]\{\boldsymbol{\delta}\} = \{\mathbf{F}\} \tag{8}$$

where $\{\ddot{\boldsymbol{\delta}}\}, \{\dot{\boldsymbol{\delta}}\}, \{\boldsymbol{\delta}\}$ are acceleration, velocity and displacement vectors, respectively. The stiffness proportional damping matrix is