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Semi-analytical skin buckling of curved orthotropic grid-stiffened shells

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ABSTRACT

With the introduction of composite materials to industrial aerospace applications, the research for innovative panel stiffening methods has gained significant interest. Possible candidates are grid-stiffened structures comprising of parallel and intersecting stiffeners forming regular polygonal patterns of skin fields. Since these thin-walled structures are critical to buckling, the structural stability is one of the driving criteria for minimum weight design. The present study investigates the local skin buckling of grid-stiffened structures known as orthogrid, isogrid, diamond grid and kagome grid with a semi-analytical Ritz energy method based on sets of trigonometric shape functions. The influence of the aspect ratio (stiffener angle), curvature and material orthotropy is shown for uni- and biaxial in-plane compression and shear. A self-stiffening effect of the grid-stiffened structures due to interaction with adjacent skin fields is identified, significantly increasing the buckling resistance of such structures. The presented results and trends support preliminary design tasks and the verification of detailed finite element analyses.

1. Introduction

In the early 1970s, General Dynamics and McDonnell Douglas layed the foundations for the grid-stiffened structures by pioneering the isogrid [11,14], an isotropic stiffened shell with ribs forming an equilateral triangular grid and having the structural panel behaviour of an isotropic material. Since then, a variety of grid-stiffened [5] and lattice structures [18] have been designed for civil and aerospace applications. However, literature on local skin buckling is rare and seems to lack a certain understanding of the fundamental behaviour of these structures. This work intents to improve the basis for this driving criterion in aerospace applications.

Today's common approach to use the finite element analysis in the early preliminary design phase shows its limits when investigating the structural stability of curved orthotropic grid-stiffened structures with arbitrary stiffener angles. The linear eigenvalue analysis appears to be highly sensitive to elementation, idealisation, mesh quality and boundary conditions. As a consequence, the results show substantial deviations when changing FE-modelling. Analytical solutions on the other hand are only available for specific flat panel shapes, loads and boundary conditions, see, e.g. [3,4,6,8,9,15,16,19–24], but even those show limited *general* agreement for local skin buckling with detailed FE models.

The currently investigated stiffener arrangements form rectangular, isosceles triangular, rhombic and tri-hexagonal patterns, denoted orthogrid, isogrid, diamond grid and kagome grid, and depicted in Fig. 1. A conventional frame-stringer architecture may thereby be considered as orthogrid with a high length-to-width or aspect ratio. The applied loads are chosen as constant in-plane compressive and shear forces per unit length. A constant curvature is defined in the lateral direction, where an infinite radius defines a flat plate. Homogeneous material orthotropy is presumed and its effects are analysed for a variable angle-ply laminate. The interaction between adjacent skin fields during buckling is mathematically incorporated and may be viewed as an elastic boundary condition on the individual skin field. A comparison with simply supported panels of rectangular and isosceles triangular shape is carried out. Henceforth, skin field denotes the elastic condition whereas single panel denotes the simply supported case. For details on the analytical fundamentals the reader may refer to textbooks at [7,12,17].

2. Analytical fundamentals

The non-linear strain–displacement relations of a thin single-curved shell following the Kirchhoff/Love [10] assumptions of zero transverse shear are described by

$$\epsilon_{xx} = u_{,x} + (w_{,x})^2/2, \quad \epsilon_{yy} = v_{,y} + w/R + (w_{,y})^2/2,$$
$$\gamma_{xy} = u_{,y} + v_{,x} + w_{,x}w_{,y}, \tag{1a}$$
$$\kappa_{xx} = -w_{,xx}, \quad \kappa_{yy} = -w_{,yy}, \quad \kappa_{xy} = -2w_{,xy}, \tag{1b}$$

where u, v, w are the displacement fields in the x, y, z direction, respectively. The shell is curved in y direction by the radius R. The comma subscript notation denotes differentiation with respect to the stated variables. The stress resultants for an orthotropic shell,

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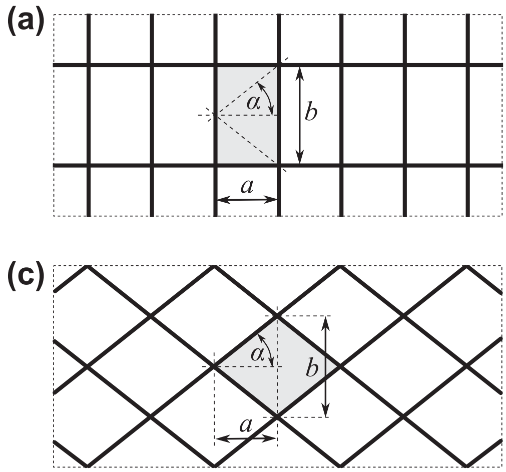

Fig. 1. Investigated stiffening pattern with skin fields of the same vertex angle, $\alpha = \arctan(b/2a)$, and defined aspect ratio, $\beta = a/b$: (a) rectangular (orthogrid), (b) isosceles triangular (isogrid), (c) rhombic (diamond grid) and (d) tri-hexagonal pattern (kagome grid).

in form of the body forces N_{xx}, N_{yy}, N_{xy} and moments M_{xx}, M_{yy}, M_{xy}, read

$$N_{xx} = A_{11}\epsilon_{xx} + A_{12}\epsilon_{yy}, \quad N_{yy} = A_{12}\epsilon_{xx} + A_{22}\epsilon_{yy}, \quad N_{xy} = A_{66}\gamma_{xy}, \tag{2a}$$

$$M_{xx} = D_{11}\kappa_{xx} + D_{12}\kappa_{yy}, \quad M_{yy} = D_{12}\kappa_{xx} + D_{22}\kappa_{yy}, \quad M_{xy} = D_{66}\kappa_{xy}, \tag{2b}$$

where A_{ij} and D_{ij} for $i, j = 1, 2, 6$ are the membrane and bending stiffnesses of the shell as defined within the Classical Laminate Theory [12]. The strain energy of this shell is derived with

$$U = \frac{1}{2} \iint \left(N_{xx}\epsilon_{xx} + N_{yy}\epsilon_{yy} + N_{xy}\gamma_{xy} + M_{xx}\kappa_{xx} + M_{yy}\kappa_{yy} + M_{xy}\kappa_{xy} \right) \, \mathrm{d}x\mathrm{d}y, \tag{3a}$$

and the potential due to the work performed by externally applied in-plane normal loads N_x, N_y and shear load N_s with

$$W = \iint \left(N_x\epsilon_{xx} + N_y\epsilon_{yy} + N_s\gamma_{xy} \right) \, \mathrm{d}x\mathrm{d}y. \tag{3b}$$

Note the definition of positive N_x, N_y representing compressive loads. By definition, only the linear strains in (1) are considered in (3b). The integration in (3) is carried out over the shell's mid-surface. The total potential Π of the shell is the sum of the strain energy potential U and the potential of external work W; that is $\Pi = U + W$.

2.1. Variational energy formulations

A body is in static equilibrium when the first variation of the total potential vanishes, $\delta\Pi = 0$. The body is critical to buckling when the total potential's second variation changes sign, that is $\delta^2\Pi = 0$. A stable solution is found from the first variation of the variational displacements in $\delta^2\Pi$. This criterion is known as the adjacent equilibrium [7] or Trefftz criterion, $\delta(\delta^2\Pi) = 0$. The original potential of the external work W vanishes from the second variation due to the defined linearity in the strains, but reappears in $\delta^2 U$. Hence, it is convenient to restate $\delta^2\Pi = \delta^2 U + \delta^2 V$, with

$$\begin{aligned} \delta^2 U = \iint \Big[& A_{11}(u_{,x})^2 + 2A_{12}(u_{,x}v_{,y} + u_{,x}w/R) + A_{22}(v_{,y} + w/R)^2 \\ & + A_{66}(u_{,y} + v_{,x})^2 + D_{11}(w_{,xx})^2 + D_{22}(w_{,yy})^2 \\ & + 2(D_{12} + 2D_{66})(w_{,xy})^2 \Big] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{4a}$$

and

$$\delta^2 V = \iint \left[N_x(w_{,x})^2 + N_y(w_{,y})^2 + 2N_s(w_{,x}w_{,y}) \right] \, \mathrm{d}x\mathrm{d}y. \tag{4b}$$

The variational operator on the displacements in (4) is omitted for brevity. Thereby, (4a) may be viewed as the virtual internal energy in the linear prebuckling configuration, whereas (4b) expresses the virtual work due to the non-linear buckling reaction. These formulations follow the principle of minimum total potential energy [12].

3. Buckling of orthotropic grid-stiffened shells

The stiffening patterns of the investigated grid-stiffened shells in Fig. 1 are generalised by defining the aspect ratio $\beta = a/b$ and the equivalent half vertex angle $\alpha = \arctan(b/2a)$. The length a and width b are the altitude and base length of the triangular panel depicted in Fig. 2. The angle α represents the inclination of the diagonal stiffeners to the x axis. Note that rhombic and tri-hexagonal unit cells would actually be of twice the length a.

A geometrical non-dimensionalisation is achieved by introducing the computational coordinates $\xi = x/a$ and $\eta = y/(b/2)$, see Fig. 2. The non-dimensional curvature is expressed by the curvature parameter $\kappa = b^2/(Rt)$ with the radius R and the thickness t, see [1]. The material properties are parameterised with $\delta_{ij} = D_{ij}/\overline{D}$, where the mean bending stiffness $\overline{D} = \sqrt{D_{11}D_{22}}$. For isotropic materials $\overline{D} \equiv D = E/(12(1-\nu^2)$ and $\delta_{11} = 1$, $\delta_{12} = \nu$, $\delta_{66} = (1-\nu)/2$ with the Young's modulus E and Poisson ratio ν. Since material homogeneity is presumed, the membrane stiffnesses are expressed in terms of the bending stiffnesses, $A_{ij} = 12D_{ij}/t^2$.

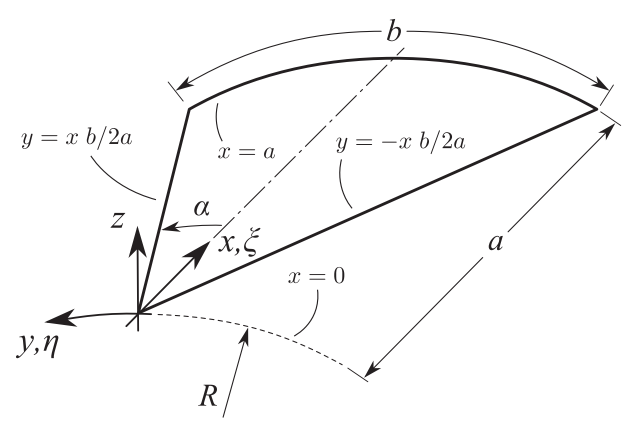

Fig. 2. Geometric properties of a triangular single-curved panel.