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An efficient shear deformation theory for free vibration of functionally graded thick rectangular plates on elastic foundation

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ABSTRACT

Free vibration analysis of plates made of functionally graded materials and resting on elastic foundation is presented by taking into account the effect of transverse shear deformations. The foundation is described by the Pasternak (two-parameter) model. The material properties of the plate are assumed to vary continuously in the thickness direction by a simple power law distribution in terms of the volume fractions of the constituents. The formulation is based on a higher order hyperbolic shear deformation theory. The equation of motion for thick functionally graded plates is obtained through the Hamilton's principle. The closed form solutions are obtained by using Navier technique and then fundamental frequencies are found by solving the results of eigenvalue problems. Accuracy of the present formulation is shown by comparing the results of numerical examples with the ones available in literature.

1. Introduction

Functionally graded material is a type of heterogeneous composite material that exhibits a continuous variation of mechanical properties from one point to another. This material is produced by mixing two or more materials in a certain volume ratio. Material properties of functionally graded material vary along the material size depending on a function. The concept of functionally graded material was first considered in Japan in 1984 during a space plane project.

Several studies have been performed to analyze the vibration of functionally graded plates. Vel and Batra [1], presented a three dimensional exact solution for free and forced vibrations of simply supported functionally graded rectangular plates. Ferreira et al. [2], analyzed the free vibrations of functionally graded plates by using a global collocation method, the first and the third-order shear deformation plate theories. Qian et al. [3], analyzed static deformations, and free and forced vibrations of a thick rectangular functionally graded elastic plate by using a higher order shear and normal deformable plate theory. Uymaz and Aydogdu [4], presented a three dimensional vibration solution for rectangular functionally graded plates. Matsunaga [5], analyzed natural frequencies and buckling stresses of plates made of functionally graded materials by taking into account the effects of transverse shear and normal deformations and rotatory inertia. Lu et al. [6], carried out free vibration analysis of functionally graded thick plates on elastic foundation based on three-dimensional elasticity. Zhao et al. [7], presented a free vibration analysis of metal and ceramic functionally graded plates by using the element-free kp-Ritz method. Chen et al. [8], analyzed the vibration and stability of functionally graded plates based on a higher-order deformation theory. Malekzadeh [9], presented the free vibration analysis of thick FG plates supported on two-parameter elastic foundation based on the three-dimensional elasticity theory. Zenkour [10], presented a sinusoidal shear deformation theory for functionally graded sandwich plates by considering the effect of rotatory inertia. Neves et al. [11,12], presented a sinusoidal shear deformation formulation and a hybrid quasi-3D hyperbolic shear deformation theory for bending and free vibration analysis of functionally graded plates.

The purpose of this study to investigate the efficiency of an improved version of a hyperbolic shear deformation theory developed by Akavci [13] for free vibration analysis of functionally grade plates. This non-polynomial higher order shear deformation theory is rather simple to use and accounts for a parabolic transverse shear deformation shape function and satisfies shear stress free boundary conditions of top and bottom surfaces of the plate without using shear correction factors. Besides, this generalized formulation can be used for the free vibration analysis of functionally grade plates by using another non-polynomial higher order shear deformation theory with the changing of the shape function. Governing equations are derived from the principle of minimum total potential energy. Navier solution is used to obtain the closed form solutions for simply supported functionally graded plates. Comparison studies are performed to verify the validity of the present results.

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2. Fundamental formulations

In the present study, a functionally graded simply supported rectangular plate which has the uniform thickness h, the length a, and the width b is considered. The geometry of the plate and coordinate system are shown in Fig. 1. The material properties of functionally graded plate are assumed to vary continuously through the thickness of the plate in according to the power law distribution as follows:

$$P(z) = P_m + (P_m - P_c)\left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{1}$$

where P denotes the effective material property like Young's modulus E and mass density ρ, P_m and P_c denotes the property of the top and the bottom faces of the plate, respectively, and p is the volume fraction exponent. The Poisson's ratio v is assumed to be constant.

Basic assumptions for the displacement field of the plate are given as below:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - zw_{0,x} + f(z)\theta_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) - zw_{0,y} + f(z)\theta_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{2}$$

where u, v, w are displacements in the x, y, z directions, u_0, v_0 and w_0 are mid-plane displacements, θ_x and θ_y are rotations of normals to the midplane about y- and x-axis and $(\cdot)_{,x}$ and $(\cdot)_{,y}$ are partial derivatives with respect to x and y, respectively. $f(z)$ represents the shape function for determining the distributions of the transverse shear strains and stresses along the thickness and given as:

$$f(z) = Tanh(rz/h) - rz/hSech^2(r/2) \tag{3}$$

The parameter "r" in Eq. (3) needs to be calculated therefore an optimization procedure is performed for determining the suitable "r" parameter by comparing the results of present theory and three dimensional solutions for a wide range of examples [1,4,14–16]. Selection of parameter "r" will be discussed in numerical results section.

Using the displacement field in Eq. (2) within the application of the linear, small-strain elasticity theory, normal and shear strains are obtained as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} - zw_{0,xx} + f(z)\theta_{x,x} \\ v_{0,y} - zw_{0,yy} + f(z)\theta_{y,y} \\ u_{0,y} + v_{0,x} - 2zw_{0,xy} + f(z)(\theta_{x,y} + \theta_{y,x}) \\ f'(z)\theta_y \\ f'(z)\theta_x \end{Bmatrix} \tag{4}$$

$$\text{where } f'(z) = \frac{df(z)}{dz} \tag{5}$$

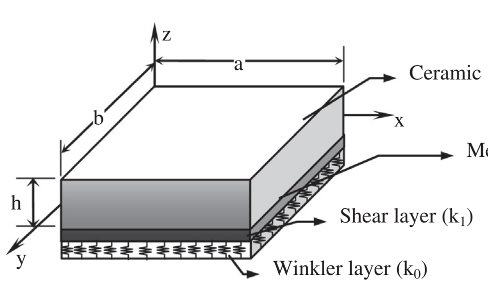

Fig. 1. Geometry and coordinates of the considered FG plate which is resting on elastic foundation.

For the functionally graded plates, the stress–strain relationships for plane-stress state can be expressed as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{6}$$

$$\text{in which}: \quad \begin{aligned} Q_{11} &= Q_{22} = E(z)/(1-\upsilon^2) \\ Q_{12} &= \upsilon E(z)/(1-\upsilon^2) \\ Q_{44} &= Q_{55} = Q_{66} = E(z)/2(1+\nu) \end{aligned}$$

3. Energy expressions and Navier solution of the vibration problem

The total potential energy of the considered functionally graded plate is expressed as:

$$\Pi = V + V_e - T \tag{7}$$

where V is the strain energy and T is the kinetic energy of FG plate and can be written as:

$$V = \frac{1}{2}\int_V (\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz})dV \tag{8}$$

$$T = \frac{1}{2}\int_V \rho(z)[(u_{,t})^2 + (v_{,t})^2 + (w_{,t})^2]dV \tag{9}$$

and V_e is potential energy of elastic foundation:

$$V_e = \frac{1}{2}\int_A \left\{k_0 w^2 + k_1\left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]\right\}dA \tag{10}$$

in which $\rho(z)$ is mass density per unit volume and $(,_t)$ represents the derivative with respect to time and k_0 and k_1 are the Winkler foundation stiffness and the shear stiffness of the elastic foundation.

Hamilton's principle is used herein to derive the equations of motion appropriate to the displacement field and the constitutive equations. The principle can be stated in analytical form as:

$$\delta\int_{t_1}^{t_2}(V + V_e - T)dt = 0 \tag{11}$$

Substituting the Eqs. (8)–(10) into the Eq. (11), then taking the variation and integrating by parts yields the following integral equation:

$$\begin{aligned} \int_{t_1}^{t_2}\Bigg[\int_V \Big[&\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{zx}\delta\gamma_{zx} \\ &-\rho(z)(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w)\Big]dv \\ &+\int_A\left[k_0 w\delta w + k_1\left(\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial y}\right)\right]dA\Bigg]dt = 0 \end{aligned} \tag{12}$$

Using the generalized displacement–strain relations (4) and stress–strain relations (6), and the fundamentals of calculus of variations and collecting the coefficients of δu, δv, δw, $\delta\theta_x$, and $\delta\theta_y$ in Eq. (12), the equations of motion are obtained as:

$$\begin{aligned} &N_{x,x} + N_{xy,y} = I_1\ddot{u}_0 - I_2\ddot{w}_{0,x} + I_4\ddot{\theta}_x \\ &N_{xy,x} + N_{y,y} = I_1\ddot{v}_0 - I_2\ddot{w}_{0,y} + I_4\ddot{\theta}_y \\ &M_{x,xx} + 2M_{xy,xy} + M_{y,yy} - k_0w_0 + k_1(w_{0,xx} + w_{0,yy}) \\ &\quad = I_1\ddot{w}_0 + I_2(\ddot{u}_{0,x} + \ddot{v}_{0,y}) - I_3(\ddot{w}_{0,xx} + \ddot{w}_{0,yy}) + I_5(\ddot{\theta}_{x,x} + \ddot{\theta}_{y,y}) \\ &P_{x,x} + P_{xy,y} - R_x = I_4\ddot{u}_0 - I_5\ddot{w}_{0,x} + I_6\ddot{\theta}_x \\ &P_{xy,x} + P_{y,y} - R_y = I_4\ddot{v}_0 - I_5\ddot{w}_{0,y} + I_6\ddot{\theta}_y \end{aligned} \tag{13}$$