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Dynamic stiffness formulation for vibration analysis of thick composite plates resting on non-homogenous foundations

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ABSTRACT

This research presents new continuous elements for thick laminated plates on a Pasternak or a non-homogeneous foundation using the Dynamic Stiffness Method (DSM). The non-homogeneous foundation consists of multi-section Winkler-type and Pasternak-type elastic foundation. The Dynamic Stiffness Matrices using the First Shear Deformation Theory (FSDT) are constructed based on the exact closed form solutions of the governing differential equations of both cross-ply and angle-ply thick composite plates on non-homogeneous elastic foundations and subjected to various types of boundary conditions. A computer program is written using the present formulation for calculating the natural frequencies and harmonic response of composite plates without contact with elastic foundation and composite plates resting on non-homogenous foundations. The validation is done by comparison of continuous element model with available results in the literature and with Finite Element Method (FEM). Different test cases confirm the advantages of the present model which is supposed to be especially efficient in dynamics analysis.

1. Introduction

The vibration of laminated composite plates on elastic foundations is of great interest to the engineering community. Usage of these systems can be found in aerospace structures, aircraft runways, nuclear reactors, building foundation slabs, railway tracks, indoor sport floors, petro-chemical and submarine structures, etc. The comprehension of static and dynamic behaviors of plates or shells resting on a non-homogenous elastic foundation is very important because such systems represent real plate-foundation structures in industry. Numerous researches are carried out in order to design safer and more economic thick laminated composite plate structures supported by non-homogenous elastic foundations. The elastic foundation can be represented by different models. The simplest model for the elastic foundation is Winkler or one-parameter model, which regards the foundation as a series of separated springs without coupling effects between each other [1]. Pasternak [2] improved this model by adding a shear layer to Winkler model. Pasternak or two-parameter model is widely used to describe the mechanical behavior of structure-foundation interactions. There have been a considerable number of studies on the plates resting on elastic foundation. Thambiratnam and Zhuge [3] solved the vibration of a stepped beam supported on a stepped elastic foundation by a finite-element method. Omurtag and Kadioglu [4] investigated the vibration of Kirchoff plates on Winkler and Pasternak foundations using a mix finite element method. Buckling and vibrations of unsymmetric laminates resting on elastic foundations under in-plane and shear forces are studied by Aiello and Ombres [5] by use of Rayleigh–Ritz method. Recently, Shen et al. [6] examined the dynamic response of laminated plates under thermo-mechanical loading and resting on a two-parameter elastic foundation by the analytical method based on Reddy's higher order shear deformable plate theory. More advance study on buckling and free vibration analysis of symmetric and anti-symmetric laminated composite plates on an elastic foundation was conducted by Akavci [7] using Navier technique and a new hyperbolic displacement model. Ugurlu et al. [8] investigated the effects of elastic foundation and fluid on the dynamic response characteristics of rectangular Kirchhoff plates using a mixed-type finite element formulation. Malekzadeh et al. [9] analyzed the vibration of non-ideal simply supported laminated plates on an elastic foundation subject to in-plane stresses with the Lindstedt–Poincare perturbation technique. The vibration of isotropic Mindlin plate on non-homogeneous Winkler foundation has been considered by Xiang [10] using Levy solutions.

The vibration analysis of structures in the medium and high frequency range plays an important role in sound transmission, sound isolation problems, fast spinning shafts as well as in the detection of defects by wave propagation or in avoiding possible resonance. Actually, only few approaches such as Statistical Energy Analysis [11] can be used efficiently for high frequency range but there is no adequate method suitable for predicting the vibration of structure in medium frequencies. Both FEM and Boundary Element

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Method (BEM) are widely used for analyzing the vibration of structure in low frequencies but they meet difficulty when dealing with the numerical computation in medium and high frequencies. In those frequency ranges, a very fine mesh of FEM or BEM model is required which can exceed the storage capacity of computer, reduce the computing time and accumulate the numerical computing errors.

The Dynamic Stiffness Method (DSM) or Continuous Element Method (CEM) [12] has been developed in order to overcome these difficulties of dynamic problems. The CEM is based on the exact closed form solution of the governing differential equations of motion which leads to the Dynamic Stiffness Matrix relating a state vector of loads to the corresponding state vector of displacements at the edges of the structure. By using CEM, one or some continuous elements are enough to compute any range of frequencies with any desired accuracy. In addition, continuous elements can also be assembled together in order to model more complex structures by using the same principle of assembly in FEM. The use of minimum of continuous elements allows a fast acquisition of harmonic response thus it reduces the computing time compared to FEM [19,20,23].

Numerous researches have been carried out for DSM of metal and composite beams [13–15] as well as for plate structures [16–18]. CEM model for vibration of isotropic shells of revolution and shells subjected to symmetrical load have also deeply been examined [19,20]. Several industrial computer codes using DSM such as BUNVIS-RG [21], PFVIBAT [22] or ETAPE [20] have been developed which confirmed the performance of CEM.

Recently, our previous study [23] focusing on DSM of composite cylindrical shells has been presented. A new study on DSM of ring structures was also introduced in [24]. Concerning composite plates, Boscolo et al. [25–27] has proposed the DSMs and the assembly of DSMs for the vibration analysis of composite plates and plates with stiffeners but in those researches only symmetric cross-ply composite plates without considering the effect of elastic foundations are investigated.

Despite of abundant researches on CEM for isotropic and anisotropic beam, shell and plate structures, to the author's best knowledge, no work related to DSM for thick composite plates including both cross-ply and angle-ply laminates resting on Pasternak foundation or non-homogenous elastic foundation has been reported in the literature. This paper aims to fill the apparent gap in this area by providing the powerful Dynamic Stiffness Matrices for the vibration analysis of thick general cross-ply and anti-symmetric angle-ply laminated plates without contact with elastic foundation and for composite plates resting on a Pasternak or a non-homogenous elastic foundation. The effects of shear forces and rotational inertia have also been taken into account. The influences of the foundation stiffness parameter, the boundary condition, the foundation length ratio and the plate thickness ratio on the frequency parameters of both general cross-ply and angle-ply composite plates are also investigated.

Our model is validated by comparing with different international researches and with FEM solutions. CEM has proved excellent accuracy, especially in the range of medium and high frequencies where FEM and BEM give unreliable solutions due to errors of meshing. Results on natural frequencies and harmonic responses of composite plates without or on non-homogenous elastic foundation make evidence the advantages of the present model: better precision of solution, size of model and computing time reduced. The proposed CEM results serve as a benchmark for FEM and other semi-analytical methods.

2. Theoretical formulations of laminated composite

Consider a thick composite laminated rectangular plate of dimensions $a \times b$ resting on a Pasternak elastic foundation as shown in Fig. 1; k_1 is linear stiffness of foundation, k_2 is the shear modulus of the sub-grade. The two opposite edges $y = 0$ and $y = b$ are assumed to be supported and boundary conditions of the two remaining edges can be any combination of free, clamped or supported types. The plate has a uniform thickness h and in general is made up of some or many laminate layers; each consists of unidirectional fiber reinforced composite material. Based on the FSDT, the displacement field at a point M_o in the middle plane of the plate is express as [28]

$$u = u_0(x,y) + z\varphi_x(x,y), \quad v = v_0(x,y) + z\varphi_y(x,y), \quad w = w_0(x,y) \tag{1}$$

where u_0, v_0 are the in-plane displacements and w_0 is the transverse displacement of a point $M_o(x,y)$ on the middle plane; φ_x, φ_y are rotations of the normal to the middle plane about y and x axes respectively. The strains are related to the displacements by the following expressions:

$$\begin{aligned}
&\varepsilon_x = \frac{\partial u_0}{\partial x} + z\frac{\partial \varphi_x}{\partial x};\ \ \varepsilon_y = \frac{\partial v_0}{\partial y} + z\frac{\partial \varphi_y}{\partial y};\ \ \gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z\left(\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x}\right)\\
&k_x = \frac{\partial \varphi_x}{\partial x};\ \ k_y = \frac{\partial \varphi_y}{\partial y};\ \ k_{xy} = \frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x};\ \ \gamma_{xz} = \frac{\partial w_0}{\partial x} + \varphi_x;\ \ \gamma_{yz} = \frac{\partial w_0}{\partial y} + \varphi_y
\end{aligned} \tag{2}$$

The constitutive equations for the composite laminated plate are determined by [28]:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix},$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{3}$$

The reduced stiffness coefficients in above equations are estimated as:

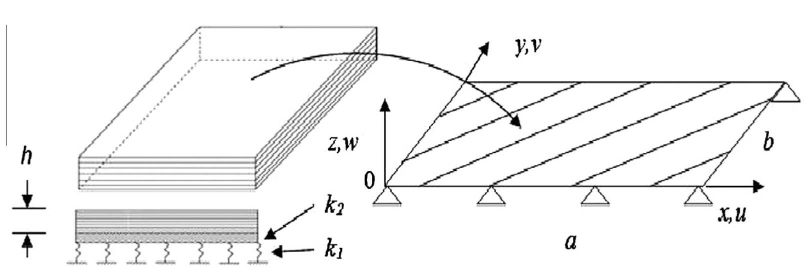

Fig. 1. A composite plate on Pasternak elastic foundation.