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Equivalent properties for corrugated cores of sandwich structures: A general analytical method

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ABSTRACT

Corrugated core sandwich structures are increasingly used as multi-functional components in many industrial areas. In order to perform efficient finite element analysis, the sandwich construction can be represented as a multi-layer two-dimensional continuum. To do so, the complex shaped core is usually represented as an orthotropic homogeneous layer. The challenge is therefore to determine the mechanical properties of the equivalent material to accurately model the sandwich structure. Several methods exist in the literature, but analytical formulations are only available for specific types of core.

In this paper, a general analytical formulation to characterize the equivalent material is proposed. The generality of the proposed approach consists in its ability to model every corrugation geometry, overcoming the main limitation of existing analytical formulations. Both beam and shell sandwich structures are modeled. Given the importance of the out-of-plane properties, all parameters for a Reissner–Mindlin representation are studied. Moreover, also non-symmetric corrugation profiles are easily processed. Thanks to its versatility, the method is validated by means of an extensive comparison with previous authors on the most common corrugation geometries. In addition, when agreement is not found on results, finite element simulations are set up to prove the precision and accuracy of the proposed formulation.

1. Introduction

1.1. Motivation

Sandwich structures have a wide applicability range in various sectors of engineering. One of the main characteristics is their high stiffness-to-mass ratio, especially under bending conditions. These structures can have different typologies of core in different materials – e.g. metallic cellular [1,2], composite corrugated [3] or metallic textile cores [4]. Several properties of the sandwich constructions, such as their dynamic or acoustic behavior, strongly depends on their core. Therefore, a proper modeling of the core is crucial in the design phase to obtain a desired response.

Among all sandwich structures, an appealing solution are the corrugated core ones. This is mainly due to their characteristic of providing good structural performance with very limited total thicknesses. Nevertheless, the small thicknesses compared to the overall dimensions affects negatively their modeling, typically made with Finite Element (FE) methods since the simulation of the three-dimensional (3D) geometry of the core requires a large number of FE elements. In order to reduce the number of elements and consequently the computational time, the complex shaped cores are typically represented as a homogeneous orthotropic layer with equivalent mechanical properties. Therefore the mechanical parameters of the material of the equivalent layer must be derived accurately, that can be usually done by means of analytical formulations or FE techniques. The main scope of this paper is to develop an analytical formulation, which is mainly based on an energetic approach, starting from the Castigliano's second theorem [5].

An important feature of the corrugated core sandwich structures is the shape of the corrugation, that can be varied for a tailored design of the components. Indeed, the core geometry strongly affects the mechanical properties of corrugated cores and thus those of the equivalent material. Several papers, available in the literature, provide analytical formulas to obtain the parameters of the equivalent material. Nevertheless, as highlighted by Cheng in [6], "the resulting complex expressions for one specific sandwich form cannot be applied to other types". This was the main motivation that pushed researchers to use FE based techniques. The proposed approach overcomes this limitation of analytical formulations for corrugated cores sandwich beams and panels by introducing a general analytical modeling technique which can be used for every corrugation geometry.

Given the importance of the transverse behavior of these structures even in simple static bending condition – not to mention

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the dynamic response –, all the parameters for a Reissner–Mindlin representation of the equivalent core material are given. Parameters are derived in both beam and plate situations. The proposed approach applies only to the core, which is the focus of this work. Nevertheless, to compare with other authors the equivalent parameters for the complete structure, thus including the two faces, they are obtained imposing conditions from the classical lamination theory. This choice is an approximation commonly done in the literature, that assumes perfect bonding between the core and the faces.

In addition, a FE based methodology is also developed to be used as reference in comparing the results from the proposed analytical approach with previous authors.

The proposed analytical method will greatly facilitate parametric studies with respect to the core geometry, as those in [7], and thus it will enable very easy design and optimization of corrugated core components [8,9]. Moreover, the results of the research can also allow the inclusion of simplified corrugated panels within vehicle concept models, which currently are based on beam-like structures [10,11].

1.2. Literature review

Several authors studied the behavior of sandwich structures, see Mackerle [12] for a deep bibliographic study up to 2001. Nevertheless, most of the papers relevant for this work were published more recently. As already introduced, the main drawback of analytical formulations, up to the proposed method, is the lack of generality, thus in the following each mentioned reference typically addresses only one corrugation geometry.

For the triangular corrugation, the most important authors are Wang and Chung [13], who descrived the main parameters to have a Reissner–Mindlin beam representation of that core and compared the accuracy with FE simulations. An interesting study on the structural performance of triangular panels was performed by Valdevit et al. [14], who indicated the critical loads for different failure mechanisms. Moreover Valdevit et al. [15] studied the possible combinations of different layers of triangular corrugations (diamond corrugations).

When considering circular and arc-and-tangent profiles, the most recent and accurate work is that by Kress and Winkler [16] in 2010, which proposed an analytical formulation for sandwich plates. In 2012 [7] they also studied the effect of geometry changes by using FE simulations. An extension of the circular profile is the core shape proposed by Yokozeki et al. [17], which has vertical segments between the two semi circumferences.

The most investigated core geometry is the trapezoidal shape, usually improperly addressed as "corrugated". Several authors were found since the pioneering study by Libove and Hubka [18]. In particular Ko [19] generalized that approach to corrugated panels with non-constant thickness. These authors investigated the complete panel, while Samanta and Mukhopadhyay [20] derived the extensional rigidity for the sole core; only the in-plane behavior was considered. Xia et al. [21] in 2012 improved that formulation obtaining a general approach valid for every corrugation profile, but still only the in-plane behavior was modeled and explicit formulas were given only for two core profiles. The same geometry was studied by Chang et al. [22] who derived analytical formulas for the bending behavior. Liew et al. [23] performed a vibrational analysis of trapezoidal and sinusoidal corrugated cores.

Another shape which has already been deeply studied in a previous work [24] is the sinusoidal corrugation. Within the most relevant authors for this shape, Nordstrand et al. [25] and Isaksson et al. [26] gave an approximation of the transverse shear moduli. Nevertheless, those formulations were found unsatisfactory and improved in [24] (the reader is referred to that paper for a comprehensive literature review on this corrugation profile).

The comparison in the following is extended to Buannic et al. [27], who performed the computation of equivalent parameters for different core shapes based on FE homogenization methods.

1.3. Naming convention for stresses

Due to the different results found in the literature on some parameters, it is worth to specify the convention used in the present work for naming stresses and strains.

The authors chose the naming convention for stress components on a 3D element that uses two subscripts: the first indicating the direction of the stress component and the second indicating the plane on which the stress component acts, i.e. the plane whose outward normal is in the indicated direction.

2. Analytical formulation

Consider a generic shaped corrugated core sandwich panel with unit width $b = 1$ (*y-direction*) and set the reference system as in Fig. 1. The corrugation can be considered periodic with period P_0 and height H_0. The corrugated lamina is supposed to have constant thickness t_c (Fig. 2). In a general case, as that in Fig. 2, the highest point of the core sample is not necessarily in correspondence of the half-period. Therefore, the curve is split in two parts having length along *x*, p_1 and p_2 respectively.

As already introduced, one of the key points which makes the proposed method very appealing is its applicability to every corrugation geometry. This is done by means of a Fourier series representation of the corrugated profile. The non-symmetry of the corrugation would lead to a Fourier series in sines and cosines. To avoid this and have a simpler formulation, the two parts of the curve are processed separately and then their contributions are combined together.

Therefore, the curves with periods $2p_1$ and $2p_2$ – obtained mirroring the two parts of the corrugation shape as shown in Fig. 3 – are considered separately. This allows them to be represented in Fourier cosine series since they are even functions.

Each curve can be represented by the function

$$f_i(x) = a_{0i} + \sum_{k=1}^{n} a_{ki} \cdot \cos\left(\frac{k\pi x}{p_i}\right) \tag{1}$$

being n the number of terms used in the series, p_i is the half-period and a_{ki} are the Fourier coefficient for the ith curve under examination ($i = 1,2$). In this paper, when numerical results are proposed, the number of terms has been chosen so that the excluded terms have a coefficient a_{ki} lower than 10^{-6}.

Another necessary function that will be used is the first derivative of the shape function:

$$f_i'(x) = -\sum_{k=1}^{n} a_{ki} \frac{k\pi}{p_i} \cdot \sin\left(\frac{k\pi x}{p_i}\right) \tag{2}$$

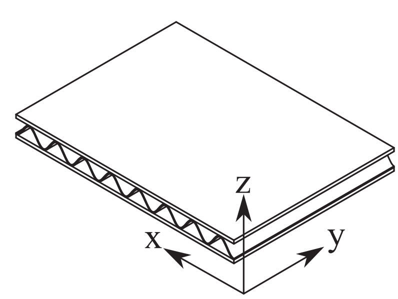

Fig. 1. Global system of reference definition.