Composite Structures 108 (2014) 779–788

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A new nonlinear high order theory for sandwich beams: An analytical and experimental investigation

Soheil Dariushi [a], Mojtaba Sadighi [b,*]

[a] Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran
[b] Composites Research Laboratory, Thermoelasticity Center of Excellence, Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Available online 21 September 2013

Keywords:
High order sandwich panel theory
Nonlinear behavior
Green strain tensor
Soft core

ABSTRACT

The paper presents a geometrically nonlinear sandwich panel theory for orthotropic sandwich beams in which the large deformation of face sheets and core is considered. The equations are derived based on high order sandwich panel theory in which the Green strain and the second Piola–Kirchhoff stress tensor are used. Nonlinear equations for a simply supported beam are derived using Ritz method in conjunction with minimum potential energy principle. The resulted set of equations is solved by the Newton–Raphson iterative technique. The results of numerical computation for beams in three point bending are presented and compared with experiments as well as with some other available results. Also simplification was used to obtain the results of linear model and in parametric studies the effect of geometric parameters on difference between results of linear and nonlinear models are discussed.

Some experimental tests on sandwich beams with glass/epoxy face sheets and soft polymeric cores were performed. The experimental results of specimens with different arrangement support the claims which were based on analytical predictions about similarities and differences between linear and nonlinear models. In all cases good agreement is obtained between the nonlinear analytical predictions and experimental results.

1. Introduction

Since 1940s that the first theoretical literatures were written in the area of sandwich construction, a large number of approaches have presented and developed. The concepts, assumptions and capabilities of the main sandwich panel theories have been described in a book by Carlsson and Kardomateas [1]. In 2005, a brilliant brief review on past, present and future of sandwich structures was presented by Vinson [2].

In 1992 Frostig et al. [3] presented a new theory based on variational principle for sandwich panels with flexible core, named high order sandwich panel theory (HSAPT). Frosting, his coworkers and other researchers used this theory to investigate sandwich structures with different shapes and materials under various loading conditions with various boundary conditions. Gradually, the HSAPT theory was improved [4–12], modified [13–17] and extended [18–20]. Geometrical nonlinearities were also considered in many literatures to increase the accuracy of HSAPT. The large number of literatures that emphasis on nonlinear HSAPT are based on kinematic relations that assume large deformations with moderate rotations for face sheets (intermediate class of deformation or Von-Karman strain) and small deformations for core (see, for example, [6,7,13,18,21–26]). Although a few references deal with large strain of the core [27,28], complete solution of equations is not provided in these articles due to complexity of solving.

In Ref. [27] Frostig et al. studied the nonlinear bending behavior of sandwich beams. The sandwich beam had a transversely flexible core with negligible in-plane flexural rigidity, including small displacement and moderate rotations in the faces and large deformation in core. HSAPT was utilized and a set of very complicated governing equations derived. To solve the equations two simplified models were employed: (1) the core was assumed to correspond to nonlinear kinematic relations of the shear angle only; (2) the core exhibits linear kinematic relations. As a numerical example, a sandwich beam loaded in three point bending was considered. The results reveal that the two simplified models were almost identical in this example.

Hohe and librescu [28] presented a nonlinear shell theory for doubly curved structural sandwich panels. The equations of motion and the corresponding boundary conditions were derived by means of Hamilton's principle. The method was similar to HSAPT. The model was employed to analyze static buckling and post-buckling of simply supported curved sandwich panels. The deformation of the sandwich shell was expressed in terms of the Green–Lagrange strain tensor and then only the nonlinear expressions with

---

* Corresponding author. Tel.: +98 21 64543448.
E-mail address: Mojtaba@aut.ac.ir (M. Sadighi).

0263-8223/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2013.09.022

respect to transverse displacements are kept while all other nonlinear terms are discarded. These assumptions lead to Von-Karman type of strains for face sheets and core. In two other papers [29,30], Hohe and Librescu used the same procedure and in the former study showed that both face sheet and core anisotropy have strong effects on different local and global buckling modes, and in the latter one reveal that compressibility of the core has a significant effect on global and local mechanical response of structural sandwich panels.

Some other relevant studies on the nonlinear behavior of sandwich panel that employed HSAPT theory are reviewed below.

Frostig [31] used HSAPT formulation to investigate the buckling behavior of sandwich panel with soft core. The kinematic relations were based on small deformation with large rotations for skins and small deformation for core. After governing the set of equations, a linearized solution approach was considered.

Sokolinsky and Frostig [32,33] applied HSAPT to analyze nonlinear buckling behavior of sandwich beam. Euler–Bernoulli beam theory with large deformations with moderate rotations for skins that have been named intermediate class of deformations were employed. The core was considered as a 2D elastic medium with small deformations. The solution approach has been explained and some numerical examples were solved and discussed. In two other papers, Frostig and Sokolinsky [34] and Sokolinsky et al. [35] used the same assumptions and procedure for buckling analysis of sandwich beams with delamination. In [35] bonding layers between face sheets and core were a non-rigid layer and the beam was subjected to external in-plane and vertical loads. In both papers [34,35] the effect of sandwich beam geometry, mechanical property and boundary conditions on critical buckling load and buckling modes were presented.

Li et al. [36] investigated buckling and postbuckling behavior of sandwich beams with delaminated faces. Two kinds of delamination were considered: a delamination within the face and a debonding between the face and the core. HSAPT was employed to govern the equations. The faces were considered as Kirchhoff–Love beams and the core was assumed that had shear resistance and is free of longitudinal stresses. The core was considered undergoing a small displacement with large rotation. It should be noted that this consideration is equal to small displacement with negligible rotations when the longitudinal stresses are assumed to be null.

Frosting and Thomsen [22,37] investigated the role of localized effects on the nonlinear behavior of sandwich panels with and without debonding. The analysis was based on HSAPT to study a sandwich beam with core material stiffened in the vicinity and far from the area of the central transverse concentrated load. The kinematic relations were corresponding to Von-Karman class of deformation. In addition, a similar work with the same assumptions and procedures but with different loading condition (uniformly distributed transverse load) presented by Frostig [38].

Li and Kardomateas [14] presented a new nonlinear HSAPT for plates in which the transverse displacement in the core is of fourth order and the in-plane displacements in the core are of fifth order in the transverse direction. The kinematic relations of face sheet were based on Von-Karman strains and the core was assumed to follow linear kinematic relations. Numerical results of this approach for a sandwich plate with orthotropic phases were validated with those obtained using the elasticity solution. In another work Li et al. [15] used the former approach to investigate the dynamic response of sandwich plates subjected to point-wise blast impact loading. The analytical results were presented as transverse deformation and stresses in the composite sandwich plate for a simply supported case.

An extended HSAPT (or EHSAPT) which considers the in-plane rigidity of core first was introduced by Carlsson and Kardomateas [1]. Phan et al. [18–20] used EHSAPT to investigate the bending [18], wrinkling [19] and buckling [20] of orthotropic sandwich beams. In [19], the face sheets undergo large displacements with moderate rotations, whereas the core strains assume to be linear and nonlinear (Von-Karman strain) in two cases. The first and linear one was a case that loads just apply on face sheets and the second and nonlinear one was a case that a uniform compressive strain through the thickness is applied to sandwich beam.

Kheirkhah et al. [39] investigated biaxial buckling of sandwich plates with a new improved HSAPT. In this approach, third order plate theory was used for face sheets, also transverse flexibility and in-plane rigidity of core have been considered. Nonlinear Von-Karman strains were used for face sheets and core. They showed that obtained overall buckling loads are in good agreement with elasticity solution and other accurate numerical results.

The main objective of this study is to present an analytical approach which considers large strains of face sheets and core. By using Green strain and second Piola–Kirchhoff stress tensor (which are energetically conjugate) and based on HSAPT, mathematical formulations are derived and then solving method is described. The analytical approach is used for numerical investigation of a simply supported sandwich beam under local transverse load. The results are compared with results of the linear HSAPT. A parametric study is done to illustrate the effect of geometrical parameters on difference between linear and nonlinear models. The differences are significant in some cases and illustrate the necessity of using nonlinear analysis to achieve reliable results. Also, to verify the analytical predictions some experimental tests were carried out. Four groups of specimens with different arrangements and sizes were made and tested. The tests were designed to illustrate the effects of geometrical parameters such as beam length, face sheets thickness and core height on the accuracy of linear and nonlinear analytical predictions.

2. Theoretical formulation

A sandwich beam with length of (L), width of (b) and total thickness of (h) is considered. Geometry of the beam and system of coordinates are shown in Fig. 1. The developed theory is based on the following assumptions:

1. (1) The face sheets and core materials are linear elastic.
2. (2) The face sheets satisfy classical beam theory with Green strains.
3. (3) The core considered as a 2D elastic medium that is flexible with respect to transverse direction and it undergoes large strains.
4. (4) The core has shear and vertical resistance and its in-plane rigidity is included (it was the novelty of EHSAPT [1]).
5. (5) The face sheets bonds to core completely.

2.1. Face sheets

Displacements in upper and lower skins due to classical beam theory assumptions are:

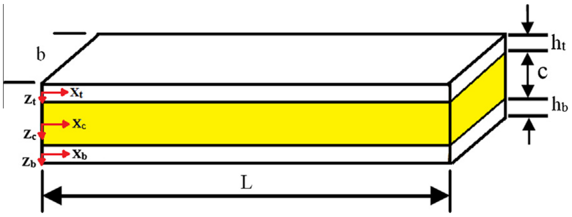

Fig. 1. Geometry and coordinates.