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Peridynamic analytical method for progressive damage in notched composite laminates

Hu Yi-le, Yu Yin *, Wang Hai

Laboratory of Civil Aircraft Structures Testing, School of Aeronautics and Astronautics, Shanghai Jiao Tong University, Shanghai 200240, China

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ABSTRACT

A new peridynamic model for fiber reinforced composite laminate was proposed and applied on the analysis of the progressive damages in composite laminate with notch or open hole. The mechanical characters of composite were described by bonds whose performance was depending on the distance and relative position of this couple of particles. In order to refine the description of composite's anisotropy, the concept of transverse modulus in classic laminate theory was introduced into the definition of bonds. Peridynamic parameters such as micromodulus and critical stretch were established following the similar concepts in elasticity theory and mechanics of composite. In this peridynamic model three types of damages in composite laminate could be analyzed: fiber fracture, matrix fracture and delamination. The mesh independency was also examined for this approach. In the examples of laminate with unidirectional and multidirectional layup, the progressive damages and failure modes were successfully analyzed under a tensile load and had a good agreement with experiments. Moreover, much more details in the procedure of damage initiations and propagations, including fiber broken, matrix fracture and delamination, could be revealed. It had shown that this peridynamic model had a great potential for composite analysis.

1. Introduction

Fiber reinforced composite had been widely used in aircraft manufacture industry because of its advance features comparing to metal material such as high strength–weight ratio, excellent corrosion and fatigue resisting performances. Various numerical methods especially the Finite Element (FE) method had been developed to analyze the progressive damages and failures in composite laminate. Among previous works, Chang et al. [1,2] had proposed one of the first two-dimensional progressive damage models for fiber reinforced composite laminates to analyze the failure process of laminates with various layups. Subsequent works [3–6] made efforts on not only the in-plane damages, but also in the delaminations employing interlayer elements or coupling adjacent layers. Although these FE models had improved the accuracy and completeness of progressive damage analysis of composites, they still had limitations in dealing with discontinuities such as crack propagations, delaminations, and fractures and penetrations. It was due to the fact that the essence of conventional FE method was the partial differential equation of displacement field, it was quite difficult to get the derivates of displacement in these discontinuous areas. Therefore the FE method suffered from the difficulties in solving discontinuous problems [7,8].

Recently, Silling [9] from Sandia National Laboratory had proposed a new continuum mechanics theory, *Peridynamic Theory*, to overcome the obstacle of discontinuities. *Bonds* were used in peridynamic theory to describe the nonlocal interactions between nodes in a finite distance. When damages emerged in materials or structures, bonds in these damaged areas would break and caused a decline in the load-carrying capacity. Peridynamic theory employed integration rather than differentiation in its motion equation so that it could avoid calculating the derivatives of displacement in discontinuous areas. Due to this feature, peridynamic method was more appropriate than FE method in analyzing the discontinuous problems. Moreover, there was no necessity to use additional failure criteria or crack growth laws as FEM did because the constitutive information of materials could be included in the peridynamic motion equation. And multiple damages and failures could occur spontaneously and freely in the model. Therefore peridynamic method could predict the progressive failure process, including damage initiations, crack propagations and final failure modes only with its motion equation.

Rapid developments on peridynamic theory had been achieved in some research fields. The theory was primitively applied in failure analysis of homogeneous materials [9–11], and then developed to heterogeneous problems such as reinforced concrete [12–14]

* Corresponding author. Tel.: +86 21 34206640.
E-mail address: yuyin@sjtu.edu.cn (Y. Yu).

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and fiber reinforced composites [15,16]. The investigations on composite structures with peridynamic theory gradually attracted more attention from researchers. Askari et al. [18] predicted the failure modes and ultimate strength of center-notched laminates under a tensile loading with 2D and 3D models using the peridynamic analysis program EMU [17]. Xu et al. [19,20] analyzed the mechanics responses of composite laminates under low velocity impact and hailstone impact, and obtained the delamination patterns, areas and residual strength under different impact energy. Kilic et al. [21,22] considered the inhomogeneous nature of composite laminate, and separated the model into fiber and matrix subregions, thus, his model could reflect the influence of fiber volume fraction. Hu et al. [23] had established an unidirectional laminate model and analyzed the dynamic loading effect on the damages of brittle materials.

For peridynamic research works on the fiber reinforced composite, the types of bonds were not adequate to model the complex anisotropic behaviors. In this paper, a new 3D peridynamic analytical model for composite laminates was proposed. To improve the depiction of composite's anisotropy, the concept of transverse modulus in classical laminate theory was introduced into the bonds' definition which allowed bonds' performance to change with the given angle between bond and fiber orientation. Properties of bonds such as *micromodulus* and *critical stretch* were deduced according to elasticity theory and classical mechanics of composites. Three types of bonds were considered in the modeling of laminates according to the difference in materials: fiber, matrix and interlayer bonds. Hence this analytical model could analyze three damage modes: fiber fractures, matrix fractures and delaminations. Based on this peridynamic laminate model, detailed procedure of progressive damages in notched laminates with different stacking sequences was analyzed under tensile loading. The deformations, crack propagation procedure and final failure modes were compared with the results from both literatures and experiments.

In addition, as the amount of bonds could easily exceed the magnitude of 10^7 in peridynamic models, the peridynamic method was relatively computational expensive [22] and usually had to using supercomputers [18–20]. In this paper, a program, PDynaComp employing the GPU [24] (Graphics Processing Unit) parallel computing technology based on CUDA [25–27] (Compute Unified Device Architecture) developed by Hu et al. [31] was used in this paper. The computing cost could be prominently reduced.

2. Peridynamic theory

Peridynamic theory [9] hypothesized that an object possessed a reference configuration R where each particle $\boldsymbol{x}$ owned a subregion R^0 with a radius δ which was called the *material horizon* as indicated in Fig. 1. Particle $\boldsymbol{x}$ interacted with the rest particles $\boldsymbol{x}'$ in subregion R^0 through a vector-valued function $\boldsymbol{f}$ which was called *pairwise force function*, a function with a dimension of force per unit volume squared. In peridynamic theory, the equation of motion at any time t could be written as follows:

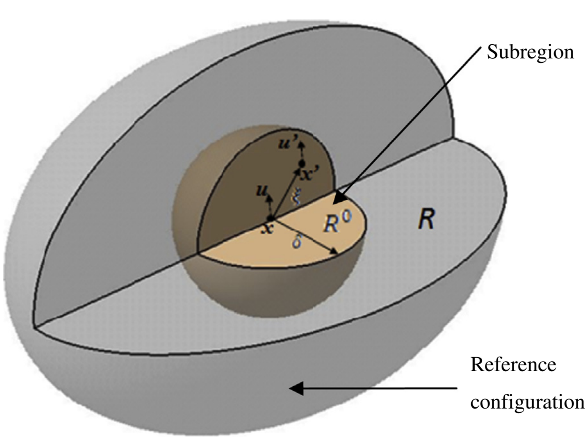

Fig. 1. Interaction between $\boldsymbol{x}$ and $\boldsymbol{x}'$.

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t)=\mathbf{L}(\mathbf{x},t)+\mathbf{b}(\mathbf{x},t) \tag{1}$$

where ρ was the density of material, $\boldsymbol{u}$ was the displacement vector, $\boldsymbol{b}$ was the external body force density, and $\boldsymbol{L}$ was the internal body force density that was given by

$$\mathbf{L}(\mathbf{x},t)=\int_{R^0}\mathbf{f}(\boldsymbol{\eta},\xi)dV_{x'}\quad \forall\mathbf{x}\in R,\;\; t\geqslant 0 \tag{2}$$

where $\boldsymbol{\eta}=\mathbf{u}'-\mathbf{u}$, $\xi=\mathbf{x}'-\mathbf{x}$ were relative displacement vectors and relative position vectors of the pairwise particles in the reference configuration, $V_{\boldsymbol{x}'}$ was the volume of particle $\boldsymbol{x}'$. Thus, the relative position vectors between $\boldsymbol{x}$ and $\boldsymbol{x}'$ in the deformed configuration could be written as $\boldsymbol{\eta}+\xi$, and the distance between two particles could be written as $y=|\boldsymbol{\eta}+\xi|$ correspondingly.

If a material satisfied the *peridynamic microelastic condition*, meaning that the net work done by interactive connections as particle $\boldsymbol{x}'$ moved along any close path was zero, the equation below could be used as its pairwise force function [10]

$$\boldsymbol{f}(\boldsymbol{\eta},\xi)=\frac{\xi+\boldsymbol{\eta}}{|\xi+\boldsymbol{\eta}|}f(y,\xi)\quad\forall\boldsymbol{\eta},\xi \tag{3}$$

where $f(y,\xi)$ was a scalar-valued function contains all constitutive information and failure definitions of the material. The scalar-valued function f depended on the relative position and distance between particle $\boldsymbol{x}$ and $\boldsymbol{x}'$. The pairwise force function of brittle materials could be defined as follows:

$$\boldsymbol{f}(\boldsymbol{\eta},\xi)=\frac{\xi+\boldsymbol{\eta}}{|\xi+\boldsymbol{\eta}|}\mu(\xi,t)cs \tag{4}$$

where c was the material's *micromodulus* function, and s was the *bond stretch* presenting the relative elongation

$$s=\frac{|\xi+\boldsymbol{\eta}|-|\xi|}{|\xi|}=\frac{y-|\xi|}{|\xi|} \tag{5}$$

In order to describe the fracture of bonds, a time-dependent function μ was introduced [10]

$$\mu(\xi,t)=\begin{cases}1 & \text{if } s(t',\xi)<s_0\\ 0 & \text{otherwise}\end{cases}\qquad t'\in(0,t)$$

where s_0 was the *critical stretch* indicating a critical value which caused breaking and decline in the load-carrying capacity of bonds. According to the definition of bonds' fracture, the damage of a particle $\boldsymbol{x}$ at some time t could be defined as

$$\phi(\boldsymbol{x},t)=1-\frac{\int_R\mu(\xi,t)dV_{x'}}{\int_R dV_{x'}} \tag{7}$$

It was showed by Eqs. (4)–(6) that all descriptions of damage or fracture were included in the definition of pairwise force function and there was no necessity to use additional criterion of strength, stress intensity factors or crack propagation regulations to achieve the solutions.

3. Numerical solution

The peridynamic method, like other meshless numerical methods, comprised neither element in the grid nor restriction of deformations between nodes. In the numerical implementation, the reference configuration was uniformly discretized into a lot of