Composite Structures 108 (2014) 848–855

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Technical Note

Hygrothermal effects on the free vibration and buckling of laminated composites with cutouts

Sundararajan Natarajan [a,*], Pratik S. Deogekar [b], Ganapathi Manickam [c], Salim Belouettar [d]

[a] School of Civil & Environmental Engineering, The University of New South Wales, Sydney, Australia
[b] Department of Civil Engineering, Indian Institute of Technology, Bombay, India
[c] Stress & DTA, IES-Aerospace, Mahindra Satyam Computer Services Ltd., Bangalore, India
[d] Henri Tudor Research Center, 29JFK, Avenue John F Kennedy, L-195 Luxembourg, Luxembourg

ARTICLE INFO

Article history:
Available online 19 October 2013

Keywords:
Vibration
Buckling
Reissner–Mindlin plate
Extended finite element method
Hygrothermal effects
Laminated composites.

ABSTRACT

The effect of moisture concentration and the thermal gradient on the free flexural vibration and buckling of laminated composite plates are investigated. The effect of a centrally located cutout on the global response is also studied. The analysis is carried out within the framework of the extended finite element method. A Heaviside function is used to capture the jump in the displacement and an enriched shear flexible 4-noded quadrilateral element is used for the spatial discretization. The formulation takes into account the transverse shear deformation and accounts for the lamina material properties at elevated moisture concentrations and temperature. The influence of the plate geometry, the geometry of the cutout, the moisture concentration, the thermal gradient and the boundary conditions on the free flexural vibration is numerically studied.

1. Introduction

Fiber reinforced laminated composites belong to a class of engineered materials that has found increased utilization as structural elements in the construction of aeronautical and aerospace vechicles, sports, as well as civil and mechanical structures. This is because of their excellent strength-to and stiffness-to-weight ratios and the possibility to tailor their properties to optimize the structural response. However, the analysis of such structures is very demanding due to coupling between membrane, torsion and bending strains; weak transverse rigidities; and discontinuity of the mechanical characteristics through the thickness of the laminates. The application of analytical/numerical methods based on various 2D theories have attracted the attention of the research community. In general, three different approaches have been used to study laminated composite structures: single layer theories, discrete layer theories and mixed plate theory. In the single layer theory approach, layers in laminated composites are assumed to be one equivalent single layer (ESL), whereas in the discrete layer theory approach, each layer is considered in the analysis. Although the discrete layer theories provide very accurate prediction of the displacements and the stresses, increasing the number of layers increases the number of unknowns. This can be prohibitively costly and significantly increase the computational time [34]. To overcome the above limitation, zig-zag models developed by Murukami [16] can satisfy the transverse shear stresses continuity conditions at the interfaces. Moreover, the number of unknowns are independent of the number of layers. Carrera and Demasi [6,17] and Carrera [5] derived a series of axiomatic approaches, coined as 'Carrera Unified Formulation' (CUF) for the general description of two-dimensional formulations for multilayered plates and shells. With this unified formulation it is possible to implement in a single software a series of hierarchical formulations, thus affording a systematic assessment of different theories, ranging from simple ESL models up to higher order layerwise descriptions. This formulation is a valuable tool for gaining a deep insight into the complex mechanics of laminated structures. The investigation of the static and the dynamic characteristics of laminated composites is fairly well covered in the literature. Existing approaches employ finite element based on Lagrange basis functions [8], meshfree methods [7,4], smoothed finite element methods [22,25] and very recently iso-geometric analysis [32,12]. The above mentioned list is no way complete, for a detailed overview interested readers are referred to [13] and references therein.

Plates with cutouts are extensively used in transport vehicle structures. Cutouts are made to lighten the structure, for ventilation, to provide accessibility to other parts of the structures and for altering the resonant frequency. Therefore, the natural frequencies of plates with cutouts are of considerable interest to designers

* Corresponding author. Address: School of Civil & Environmental Engineering, The University of New South Wales, Sydney, NSW 2052, Australia. Tel.: +61 2 93855030.

E-mail address: sundararajan.natarajan@gmail.com (S. Natarajan).

0263-8223/$ - see front matter
http://dx.doi.org/10.1016/j.compstruct.2013.10.009

of such structures. Most of the earlier investigations on plates with cutouts have been confined to isotropic plates [23,1,11] and laminated composites [29,30]. Moreover, the laminated composites may be subjected to moisture and temperature environment during its service life. The moisture concentration and thermal environment can have significant impact on the response of such laminated structures. Whitney and Ashton [33] employed Ritz method to analyze the effect of environment on the free vibration of symmetric laminates. Patel et al. [24] employed shear flexible Q8 quadrilateral element to study the hygrothermal effects on the structural behavior of thick composite laminates. Patel et al. employed higher order accurate theory and studied the importance of retaining higher order terms in the displacement approximation. The effect of cutouts on the buckling behavior of laminated composites were studied in [21,28]. And more recently, Aydin Komur et al. [14] and Ghannadpour et al. [10] studied the buckling behavior of laminated composites with circular and elliptical cutouts using the finite element method and first order shear deformation theory. Their study was restricted to a limited number of configurations, because the mesh has to conform to the geometry. Moreover, to the author's knowledge the effect of cutout on the free vibration and buckling behavior of laminated composites in hygrothermal environment has not been studied earlier or was limited to simple configurations. In this study, we present a framework that provides flexibility to handle internal discontinuities. Without loss of generality, we only present the results for standard cutouts. Interactions of cutouts and cracks emanating from cutouts can be handled within this framework. For more details, interested readers are referred to [26] and references therein.

In this paper, we study the influence of a centrally located cutout on the fundamental frequency and the critical load of multilayered composite laminated plates in hygrothermal environment. Both circular and elliptical cutouts are considered for the study. A structured quadrilateral mesh is used and the cutouts are modelled independent of the mesh within the extended finite element (XFEM) framework. A systematic parametric study is carried out to bring the effect of the boundary conditions, the thermal gradient ΔT, the change in moisture concentration ΔC, the geometry of the cutout on the free flexural vibration and buckling of laminated composites.

2. Theoretical formulation

Using the Mindlin formulation, the displacements u, v, w at a point (x, y, z) in the plate (see Fig. 1) from the medium surface are expressed as functions of the mid-plane displacements u_o, v_o, w_o and independent rotations β_x, β_y of the normal in yz and xz planes, respectively, as

$$\begin{aligned} u(x,y,z,t) &= u_o(x,y,t) + z\beta_x(x,y,t) \\ v(x,y,z,t) &= v_o(x,y,t) + z\beta_y(x,y,t) \\ w(x,y,z,t) &= w_o(x,y,t) \end{aligned} \tag{1}$$

where t is the time. The strains in terms of mid-plane deformation can be written as:

$$\boldsymbol{\varepsilon} = \left\{ \begin{array}{c} \boldsymbol{\varepsilon}_p \\ \mathbf{0} \end{array} \right\} + \left\{ \begin{array}{c} z\boldsymbol{\varepsilon}_b \\ \boldsymbol{\varepsilon}_s \end{array} \right\} - \{\bar{\boldsymbol{\varepsilon}}_o\} \tag{2}$$

The midplane strains $\boldsymbol{\varepsilon}_p$, the bending strains $\boldsymbol{\varepsilon}_b$ and the shear strains ε_s in Eq. (2) are written as:

$$\boldsymbol{\varepsilon}_p = \left\{ \begin{array}{c} u_{o,x} \\ v_{o,y} \\ u_{o,y} + v_{o,x} \end{array} \right\}, \quad \boldsymbol{\varepsilon}_b = \left\{ \begin{array}{c} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{array} \right\}, \quad \boldsymbol{\varepsilon}_s = \left\{ \begin{array}{c} \beta_x + w_{o,x} \\ \beta_y + w_{o,y} \end{array} \right\} \tag{3}$$

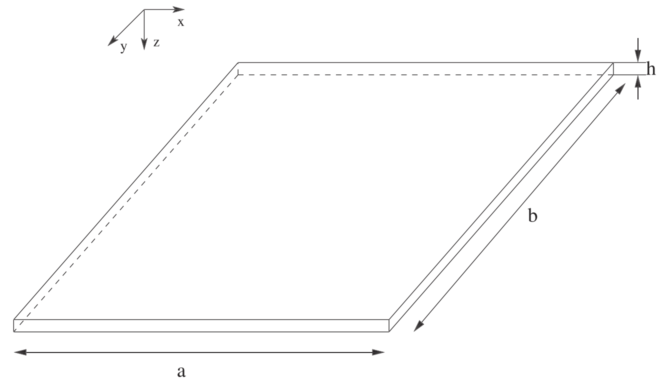

Fig. 1. Coordinate system of a rectangular laminated plate.

where the subscript 'comma' represents the partial derivative with respect to the spatial coordinate succeeding it. The strain vector $\{\bar{\boldsymbol{\varepsilon}}_o\}$ due to temperature and moisture is represented as:

$$\bar{\boldsymbol{\varepsilon}}_o = \left\{ \begin{array}{c} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\varepsilon}_{xy} \end{array} \right\} = \Delta T \left\{ \begin{array}{c} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{array} \right\} + \Delta C \left\{ \begin{array}{c} \gamma_x \\ \gamma_y \\ \gamma_{xy} \end{array} \right\} \tag{4}$$

where ΔT and ΔC are the rise in temperature and the moisture concentration, respectively. α_x, α_y and α_{xy} are the thermal expansion coefficients in the plate coordinates and can be related to the thermal coefficients (α_1, α_2 and α_3) in the material principal directions and γ_x, γ_y and γ_{xy} are the moisture expansion coefficients similar to thermal expansion coefficients in the plate coordinates. The constitutive relations for an arbitrary layer k in the laminate (x, y, z) coordinate system can be expressed as:

$$\boldsymbol{\sigma} = \left\{ \begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{array} \right\} = [\overline{Q}_k] \left\{ \left\{ \begin{array}{c} \boldsymbol{\varepsilon}_p \\ \mathbf{0} \end{array} \right\} + \left\{ \begin{array}{c} z\boldsymbol{\varepsilon}_b \\ \boldsymbol{\varepsilon}_s \end{array} \right\} - \{\bar{\boldsymbol{\varepsilon}}_o\} \right\} \tag{5}$$

where the terms of $[\overline{Q}_k]$ matrix of kth ply are referred to the laminate axes and can be obtained from the $[Q_k]$ corresponding to the fiber directions with the appropriate transformations. The governing equations are obtained by applying Lagrangian equations of motion:

$$\frac{d}{dt}\left[\frac{\partial(T-U)}{\partial \dot{\boldsymbol{\delta}}_i}\right] - \left[\frac{\partial(T-U)}{\partial \boldsymbol{\delta}_i}\right] = 0, \quad i = 1, \ldots, n \tag{6}$$

where T is the kinetic energy, given by:

$$T(\boldsymbol{\delta}) = \frac{1}{2}\int_\Omega \{p(\dot{u}_o^2 + \dot{v}_o^2 + \dot{w}_o^2) + I(\dot{\beta}_x^2 + \dot{\beta}_y^2)\} \mathrm{d}\Omega \tag{7}$$

where $p = \int_{-h/2}^{h/2} \rho \,\mathrm{d}z$, $I = \int_{-h/2}^{h/2} z^2 \rho \,\mathrm{d}z$ and ρ is the mass density of the plate. The strain energy function U is given by:

$$U(\boldsymbol{\delta}) = \frac{1}{2}\iint \left[\sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} \boldsymbol{\sigma}^{\mathrm{T}} \boldsymbol{\varepsilon} \,\mathrm{d}z\right] \mathrm{d}x\,\mathrm{d}y \tag{8}$$

where $\boldsymbol{\delta} = \{u, v, w, \beta_x, \beta_y\}$ is the vector of the degrees of freedom associated to the displacement field in a finite element discretization. Substituting Eqs. (7) and (8) in Lagrange's equations of motion with the constitutive equations and following the procedure given in [27], the following discretized equation is obtained:

$$\mathbf{M}\ddot{\boldsymbol{\delta}} + [\mathbf{K} + \mathbf{K}_R + \mathbf{K}_G]\boldsymbol{\delta} = \mathbf{f}_T \tag{9}$$

where $\mathbf{K}$ is the global linear stiffness matrix, $\mathbf{K}_R$ and $\mathbf{K}_G$ are the global geometric stiffness due to the residual stresses and the applied in-plane mechanical loads, respectively, $\mathbf{M}$ is the global mass matrix and $\mathbf{f}_T$ is the global hygrothermal load vector. After substituting the