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Nonlinear vibrations and multiple resonances of fluid filled arbitrary laminated circular cylindrical shells

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ABSTRACT

Nonlinear forced vibrations of water-filled, laminated circular cylindrical shells are studied by using the Amabili–Reddy nonlinear higher-order shear deformation theory and energy approach in the Lagrangian description. The fluid is modeled by potential flow. It is assumed that the shell is subjected to a steady harmonic concentrated force acting in the radial direction. Pseudo arc-length continuation and collocation technique is used to carry out bifurcation analysis and to obtain nonlinear frequency–amplitude responses. Direct time integration of the equations of motion has also been performed by using Gear's backward differentiation formula (BDF) to obtain time histories, phase space diagrams and Poincaré maps. The effect of internal fluid and lamination angle on the frequency–amplitude response in the neighborhood of the resonance frequency, traveling wave solution and internal resonances of simply supported shells are investigated. It is shown that water-filled composite shells may exhibit complex nonlinear dynamics including a rare and intricate 1:1:1:1 internal resonance.

1. Introduction

The continuous development of material science along with the increasing demands for producing structures with optimum weight and stiffness has led to the use of advanced materials (e.g. laminated composites) in designing and manufacturing shell type structures. Among different fields of application, aerospace applications are particularly challenging as they involve fluid–structure interactions and the use of composite materials. In fact, models need to take into account (i) nonlinear effects such as large structural deflections, and (ii) fluid–structure interaction. One critical example of such challenging applications is the super-lightweight tanks in spacecrafts (e.g. the NASA space shuttle).

An extensive literature review on the nonlinear dynamics of shells in *vacuo*, filled with or surrounded by quiescent and flowing fluid has been provided by Amabili and Païdoussis [1]. The monograph of Amabili [2] has also provided a comprehensive review on nonlinear vibrations and dynamic stability of shells.

Systematic research on nonlinear dynamics and large-amplitude vibrations of circular cylindrical shells with quiescent and flowing fluid has been carried out by Amabili et al. [3–7] and Pellicano et al. [8]. In 2003, Amabili [9,10] extended his previous works to nonlinear forced vibrations of imperfect, simply supported empty and fluid-filled circular cylindrical shells subjected to concentrated harmonic force. In particular, a full series of comparisons between numerical and experimental results for large-amplitude vibrations of fluid-filled, imperfect shells were performed in [9], and comparisons between numerical results obtained from Donnell's nonlinear theory (with and without in-plane inertia), Sanders–Koiter, Flügge–Lur'e-Byrne and Novozhilov's theories for water-filled shells were conducted in [10]. Karagiozis et al. [11] developed two numerical models based on Donnell's nonlinear shell theory with and without in-plane inertia to study nonlinear vibrations of clamped fluid-filled shells. In a series of papers Koval'chuk and Kruk [12], Koval'chuk et al. [13] and Kubenko et al. [14] discussed the problem of nonlinear forced vibrations of completely filled simply supported circular cylindrical shells by using Donnell's nonlinear shallow shell theory and the Krylov–Bogolyubov–Mitropol'skii averaging technique.

Reduced-order models based on (i) proper orthogonal decomposition (POD) and (ii) the nonlinear normal modes method have been obtained in Refs. [15–18] to study large-amplitude vibrations of fluid-filled shells subjected to radial harmonic excitation. In particular, Amabili et al. [15,16] compared the Galerkin and POD models for a water-filled circular cylindrical shell. Moderate and large-amplitude vibrations were investigated and results showed that the POD allows for a drastic reduction of the computational effort. Touzé and Amabili [17] used the nonlinear normal modes method and built a reduced-order model with only two-degrees of freedom that showed accurate results for vibration amplitudes up to twice the thickness of the water-filled shell. The detailed

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comparison between POD and nonlinear normal modes method in studying nonlinear vibrations of fluid-filled shells has been given by Amabili and Touzé [18].

Nonlinear free and forced vibrations of orthotropic and composite circular cylindrical shells have been studied by Jansen [19–22] using Donnell's nonlinear shell theory and a semi-analytic method. Nonlinear forced vibrations of symmetrically laminated composite shells have been investigated by Amabili and Reddy [23] and Amabili [24,25] based on a new higher-order shear deformation theory that takes into account nonlinear terms involving both the normal and in-plane displacements. In particular, Amabili and Reddy [23] found that the conventional higher-order shear deformation theory with von Kármán type nonlinearities gives inaccurate results for vibration amplitudes of about twice the shell thickness. Amabili [24] compared the nonlinear frequency response curves of simply supported shells obtained by (i) higher-order shear deformation theory with von Kármán type nonlinear terms, (ii) Novozhoilov shell theory, and (iii) the new theory of Amabili–Reddy. He mentioned that the conventional higher-order shear deformation theory only gives accurate results for modes with high circumferential wave-numbers, Novozhilov theory gives acceptable results for thin laminated shells and the new theory of Amabili and Reddy is preferable, if nonlinear vibrations of very thick shells are of interest. Amabili also investigated the rich nonlinear dynamics and internal resonances of symmetric cross-ply circular cylindrical shells in Ref. [25] and nonlinear vibrations of angle-ply circular cylindrical shells in Ref. [26]. The new theory of Amabili and Reddy has also been successfully used in studying nonlinear vibrations of functionally graded shallow shells [27] and laminated cylindrical panels [28].

Fluid–structure interaction has been rarely studied in the literature for the large-amplitude vibrations of composite shells. The effect of material orthotropy on the nonlinear vibrations and dynamic instability of circular cylindrical shells in contact with flowing fluid have been examined by del Prado et al. [29]. Donnell's nonlinear shell theory was used and the fluid was assumed to be non-viscous and incompressible. The equations of motion were obtained by Galerkin approach and were solved by a Runge–Kutta technique. An eight-degree of freedom model was considered that included the driven, companion, gyroscopic, and four axisymmetric modes. Lakiza [30] studied the nonlinear vibrations of fluid-filled glass-fiber reinforced cylindrical shells subjected to radial two-frequency excitation.

Composite shell theories have also been used to model arteries conveying flow in biomechanics. As a first attempt to study the nonlinear response of human aortic segments, Amabili et al. [31] used a laminated circular cylindrical shell model and nonlinear higher-order shear deformation theory to study the nonlinear stability of an aortic segment under steady flow conditions. The aorta was modeled as a three layer composite shell and the fluid model contained the unsteady effects of linear potential flow theory and the steady viscous effects obtained from the time-averaged Navier–Stokes equations. It was shown that, the aortic segment loses stability by divergence with deformation of the cross section at a critical flow velocity for a given static pressure, exhibiting a strong subcritical behavior with partial or total collapse of the inner wall.

The present paper aims to extend the recent work of Amabili [26] in studying nonlinear forced vibrations of angle-ply shells, by taking into account the effect of internal fluid, different lamination angles, and the possibility of skewed modes (modes with nodal lines not parallel to the longitudinal axis) in studying frequency–amplitude responses, traveling wave solutions and internal resonances. The new nonlinear high-order shear deformation theory of Amabili and Reddy has been used to study the geometrically nonlinear vibrations of water-filled laminated shells with multiple generally oriented orthotropic layers. Modal damping is introduced to simulate dissipation and a concentrated harmonic force is assumed to act in the radial direction. Simply supported movable boundary conditions are considered and the energy functional is reduced to a system of nonlinear ordinary differential equations with quadratic and cubic nonlinear terms using admissible functions that satisfy geometric and natural boundary conditions identically. A code based on pseudo arc-length continuation and collocation technique is used to perform a bifurcation analysis. Direct time integration of the equations of motion using Gear's backward differentiation formula (BDF) has also been performed to obtain time histories, phase space diagrams and Poincaré maps. Results reveal that internal fluid enhances the strength of the softening type response while lamination angle plays different roles depending on the type of laminate. In fact, it is observed that multi-layer laminates with generally oriented orthotropic layers represent stronger nonlinearity in comparison to angle-ply laminates and anti-symmetric cross-ply shells. Furthermore, it is shown that water-filled composite shells may exhibit complex nonlinear dynamics including a rare 1:1:1:1 internal resonance.

2. Equations of motion

2.1. Shell kinematics

Fig. 1 shows an arbitrary laminated circular cylindrical shell made of a finite number of orthotropic layers, with length L, uniform thickness h and mean radius R in cylindrical coordinates (x, θ, z). According to the Amabili–Reddy higher-order shear deformation theory [23], the displacements of a generic point on the shell are as follows

$$u_1 = u + z\phi_1 + z^2\psi_1 + z^3\gamma_1 + z^4\theta_1, \tag{1a}$$

$$u_2 = (1 + z/R)v + z\phi_2 + z^2\psi_2 + z^3\gamma_2 + z^4\theta_2, \tag{1b}$$

$$u_3 = w, \tag{1c}$$

where u, v and w are the displacements of a point on the middle surface in x, $y = R\theta$ and z directions respectively, and ϕ_1 and ϕ_2 are rotations of the transverse normals about y and x axis, respectively. $\psi_1, \psi_2, \gamma_1, \gamma_2, \theta_1, \theta_2$ are functions to be determined in terms of u, v, w, ϕ_1 and ϕ_2, which are the 5 parameters that describe the shell deformation. The expressions for these functions can be found in Ref. [23].

The strain components in the higher-order shear deformation theory are

$$\varepsilon_{xx} = \varepsilon_{x,0} + z(k_x^{(0)} + zk_x^{(1)} + z^2k_x^{(2)}), \tag{2a}$$

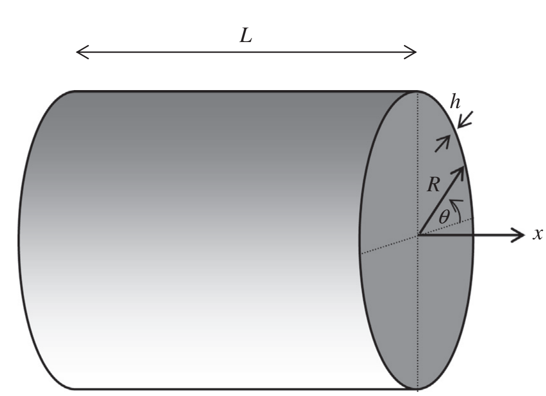

Fig. 1. Circular cylindrical shell: coordinate system.