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Analytical modeling of indentation of composite sandwich beam

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ABSTRACT

This paper deals with static indentation of sandwich beams with a foam core. An analytical model is presented assuming that the specific response of foams in compression can be assimilated to an elastic–perfectly plastic behavior. The elastic part is represented using Vlasov's model. The displacements and stress calculated with this two parameters model are compared with results given by Winkler's theory and Finite Element Method. Vlasov's idealization gives more accurate results that Winkler's model. A complete study of the influence of the parameters of Vlasov's model is performed. Then, plasticity is added to the model to represent the non-linear response of the core. The load–displacement response of sandwich beams under static indentation is calculated and compared to experimental and Finite Element results. A good correlation is found. The size of the area where the foam is crushed given by the developed model is in good agreement with Finite Element analysis.

1. Introduction

This article deals with the analytical modeling of indentation of composite sandwich beam with composite face sheets and foam cores. Composite sandwich structures are widely used in the field of transportation (helicopter blades, ship's hull, etc.) for their low weight and high in plane and flexural stiffness. During their life, these structures can be subjected to localized loadings, like impacts of birds or hailstones. Even though a visual examination of the loaded surface may reveal little damage, significant damage might exist and cause important reductions in the strength of the structure. That is why significant research efforts have been focused on the study of impacts on sandwich structures and the study of the local deformations in the contact zone is an important component of it.

A comprehensive review article of Abrate [1] discusses impacts on composite sandwich structures. The contact laws for sandwich structures differ significantly from that for a monolithic laminates. The local deformation in the contact zone consists of the local indentation of the top face sheet and in a large part of the deformation of the core material under that face sheet. Many experimental studies have been carried out [2–7] to understand the behavior of sandwich beams under quasi-static and dynamic loadings. Failure modes have been identified. For low impact energy levels, the beams deform within the elastic range. For higher energy levels, four basic modes are identified: core crushing, facesheet buckling, delamination within the facesheet and debonding between the facesheet and the core. The contact force-indentation relation for quasi-static and low velocity impacts is non-linear, owing to the specific response of the foam core or honeycomb core in compression [8,9].

In order to predict the overall response of the sandwich structures, several models have been proposed. The Finite Element Method is often used [10–13] to study the static and dynamic indentation of sandwich structures. The developed models are able to well represent the static and dynamic indentation response of sandwich panels. The computed failure modes match experimental observations for several impact energies, impact angles and specimen configurations. However, setting up such models is complicated and calculation times are high.

Many analytical models have been developed to predict indentation failure in sandwich structures. The comprehensive review of Wang et al. [14] discusses on the various idealizations developed to model beams and plates on elastic foundations. The modeling of the core generally follows one of two main strategies. The first one consists of modeling the core using linear elastic springs. These springs are independent so shearing effects are not taken into account [15,16]. This model has been modified to introduce interactions among the spring elements by incorporating a thin elastic membrane subjected to a constant tension (Filonenko-Borodich model [17]); an elastic beam or plate (Heteneyi model [18]); or a beam or a plate that only undergoes transverse shear deformations (Pasternak model [19]). One of the major disadvantages of these models is that the identification of the parameters of the

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foundation is very difficult. Indeed, as shown by Gdoutos et al. in [15], the stiffness value of the foundation has to be deduced from experimental results. In addition, Heteneyi [18] shows that the springs' stiffness expression depends on the loading conditions.

The second modeling strategy consists of idealizing the core as a three-dimensional continuous elastic solid. From this approach, Vlasov [20] developed a two parameter model of the core assuming that there is no horizontal displacements and that vertical displacements can be represented by a single shape function. The main advantage of this method is that, contrary to one parameter idealizations, the stiffness parameters can be easily deduced from material data, regardless of the loading conditions. Moreover, no specific experimental identification is necessary. This idealization has been used to model indentation on sandwich beams [21] and on sandwich panels [22], but only within the elastic range.

To reproduce the non-linear behavior of a sandwich beam, Zenkert et al. [23] presents a modified Winkler model in which the core is represented by a perfect elastic plastic material law. Experimental load–displacement curves for indentation tests are matched by the model, but the model contains rather rough approximations. Firstly, transverse shear stresses, that play an important role in the damage of sandwich structures, are not accounted for. Secondly, the foundation modulus is geometry dependant and can only be extracted by quite complex experimental procedures.

In the present paper, the core is modeled using Vlasov's theory, by a two parameter elastic–perfectly plastic foundation. This modeling strategy presents three main advantages. Firstly, the model is able to reproduce the linear and non-linear response of the sandwich beam. Secondly, transverse shear in the foam is taken into account for a better accuracy of the stress state in the sandwich beam. Thirdly, the parameters of the model can be found only from material data: no specific experimental identification is necessary.

The calculated behavior of the beam before the crushing of the core is compared with results given by the Winkler theory and Finite Element Method. Contrary to Winkler's model, the deflection of the skin and the stresses in the beam calculated with Vlasov's model correlate well with FEM results. The influence of the parameters of the Vlasov modeling on the response of a sandwich beam is studied. Then, the non-linear behavior of the core is taken into account to predict the size of the crushed zone and the contact force response. Results are compared to Finite Element and experimental results. A good correlation is found for the indentation load–deflection curve and for the size of the area where the foam is crushed.

2. Linear elastic response

Local loading on sandwich beams can be studied analytically by considering the core as an elastic foundation. In this section we recall the results obtained with Winkler's model and develop a two parameter model for the foundation using Vlasov's approach.

2.1. Winkler's model

The one parameter Winkler foundation model is widely used to study local indentation of sandwich beams. For the sake of completeness, Winkler's formulation is presented here. This model represents the core as a system of independent and linear elastic springs (Fig. 1).

The pressure–deflection relation is given by:

$$p = kw \tag{1}$$

where p is the pressure response of the core, k the stiffness of the springs and w the deflection of the facesheet. For a localized loading the governing equation of the skin is:

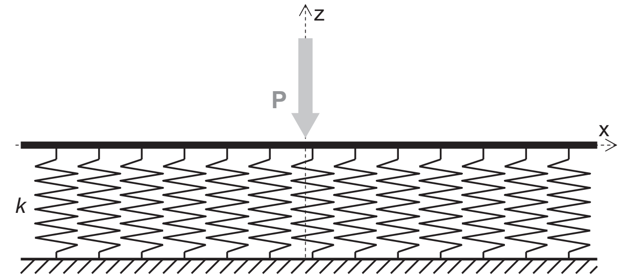

Fig. 1. Beam resting on one parameter Winkler elastic foundation.

$$E_sI_s\frac{d^4w}{dx^4} + kw = 0 \tag{2}$$

where E_sI_s is the flexural rigidity of the skin. The parameter k is the only parameter that represents the behavior of the whole core. Its value must be determined with care. In the models described in [23], k is roughly estimated as:

$$k = \frac{E_cb}{t_c} \tag{3}$$

where E_c is the core Young's modulus, b the width of the beam and t_c the core thickness. The stiffness due to shearing effects is neglected and this estimation may be inappropriate in some cases.

The value of k can also be identified experimentally. Gdoutos et al. [15] gives an estimated expression of k, function of the skin and core moduli (respectively E_s and E_c) and of the facing thickness t_s:

$$k = 0.64\frac{E_c}{t_s}\left(\frac{E_c}{E_s}\right)^{1/3} \tag{4}$$

The solution of Eq. (2) is:

$$w(x) = e^{-\lambda x}(A\sin(\lambda x) + B\cos(\lambda x)) + e^{\lambda x}(C\sin(\lambda x) + D\cos(\lambda x))$$
$$\text{with} \quad \lambda = \left(\frac{k}{4E_sI_s}\right)^{1/4} \tag{5}$$

The constants A, B, C and D are determined from boundary conditions:

– the skin remains bounded at infinity.

$$\lim_{x\to\infty} w(x) = 0 \quad \text{and} \quad \lim_{x\to\infty}\frac{dw}{dx}(x) = 0 \tag{6}$$

– the slope at the origin is zero (symmetry).

$$\frac{dw}{dx}(0) = 0 \tag{7}$$

– the shear force at the origin is half the force P applied.

$$-\frac{P}{2} = V(0) = -E_sI_s\frac{d^3w}{dx^3}(0) \tag{8}$$

Thus, the elastic solution can be written as:

$$w(x) = \frac{P}{8\lambda^3E_sI_s}e^{-\lambda x}(\sin(\lambda x) + \cos(\lambda x)) \tag{9}$$

2.2. Vlasov's model

The most important weakness of Winkler idealization is the independence of the springs. A first estimation of their stiffness must be done experimentally and the applied loads get localized.