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Probabilistic analysis and optimization of energy absorbing components made of nanofiber enhanced composite materials

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ABSTRACT

An enhanced thermoset polymer matrix with randomly distributed carbon nanofibers (CNFs) is combined with conventional long fibers to form a hybrid composite material for application to impact energy absorbing components. The multi-inclusion method in combination with functionally graded interphase is used for stiffness characterization and shear-lag theory combined with quasi-isotropic lamination approximation is used for strength prediction. Axial crush simulations are performed using MD Nastran with a micromechanics-based progressive failure analysis constitutive model. The stochastic uncertainties in the geometric and material properties of CNF as well as the three-dimensional, non-homogeneous CNF–matrix interphase are represented using probability theory. Through Monte Carlo simulations, these uncertainties are propagated to the homogenized macroscopic properties of the nano-enhanced matrix and subsequently to the stiffness and strength properties of the composite laminate as well as the energy absorbing characteristics of the crush tube. A probabilistic design optimization problem is formulated for minimizing the failure probability associated with the specific energy absorption of the composite tube. A dual surrogate modeling approach is used for approximating the failure probability and solving the optimization problem using sequential quadratic programming. The modeling approach, uncertainty analysis, and probabilistic optimization results are presented and discussed.

1. Introduction

Studies over the past two decades have shown the potential improvement in properties and performance of multiscale (hybrid) composites in which nano and microscale particles are incorporated into fiber-reinforced polymer matrix materials [1–3]. Diversity of the length scales and mechanical properties of the individual constituents provide opportunities for research in the modeling, analysis, and design optimization of structural components made of hybrid composite materials.

Researchers have experimentally studied the energy absorption in composite components by varying design parameters, such as the type, size or volume fraction of filler materials [4–6]. However, because of the anisotropic properties and morphology of the nanoparticles, it is difficult to intuitively predict the energy absorption properties of the resulting nanocomposites [7]. Thus, it is desirable to conduct analytical or numerical analyses to understand how nanoparticles affect the mechanical behavior of the composite material. In addition, design of a structural component using nanocomposite materials is more complicated than one made of conventional materials due to the complex behavior of the material and the physical interactions among the material constituents. Thus, finding an optimum design extends beyond enhanced structural geometry to global or local tailoring of material properties [8].

In addition, the wide-ranging morphology of the nanoreinforcements (nanoparticulates, nanofibers, nanotubes, nanoclays, etc.) can exhibit a great degree of uncertainty/variability [9–14]. In the case of carbon nanotube (CNT) and carbon nanofiber (CNF), for example, variability could include the length, diameter, waviness, as well as the end configuration (end-caps). There is also uncertainty in the geometric and material properties of the interphase region between the nanoreinforcements and the matrix [15]. Such uncertainties can propagate to the overall properties of the resulting composite material and consequently affect the design of composite structures. Hence, a probabilistic analysis and optimization approach is necessary in design of components made of hybrid composite materials.

In this work, the uncertainties in both CNF-enhanced composite material and structure are represented using probability distributions and propagated to the response characteristics of a structural

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component under crush load. A probabilistic design optimization problem is formulated and solved, where the effect of stochastic (inherent) uncertainties on the energy absorption is modeled in calculating the failure probability. Fig. 1 shows the general framework used in representation and propagation of the uncertainties stemming from material and structural sources with application in design under uncertainty.

2. Stiffness and strength modeling of nano-enhanced matrix

A multi-inclusion (MI) model [16] in combination with a piecewise constant approximation for functionally graded interphase is used to calculate the stiffness properties of nanoreinforced matrix material. The stiffness calculation approach and modeling of the effect of interphase on the elastic properties of nanocomposite follow that presented in Rouhi et al. [17,18] and Rouhi [19]. In summary, considering the interphase representation in Fig. 2, the overall elastic tensor of the multi-phase region is treated as the volumetric average of properties of the individual constituent materials given by

$$\bar{\mathbb{L}}^{MI} = \mathbb{L} : \left\{ \mathbb{I} + \sum_{\alpha=1}^{n} \mathbb{f}_\alpha (\mathbb{S}^\Omega - \mathbb{I})^{-1} \right\} : \left\{ \mathbb{I} + \sum_{\alpha=1}^{n} \mathbb{f}_\alpha \mathbb{S}^\Omega : (\mathbb{A}^\alpha - \mathbb{S}^\Omega)^{-1} \right\}^{-1} \quad (1a)$$

$$\mathbb{A}^\alpha \equiv (\mathbb{L} - \mathbb{L}^\alpha)^{-1} : \mathbb{L} \quad (1b)$$

where $\mathbb{L}$ is the fourth-order stiffness tensor, $\mathbb{I}$ is the fourth-order identity tensor, $\mathbb{f}_\alpha$ is the volume fraction of the α interphase region, and $\mathbb{S}$ is the fourth-order Eshelby tensor that also depends on the geometry of the MI model.

Using the functionally graded representation of the interphase as shown in Fig. 2, each elastic property (e.g., Young's modulus, Poisson's ratio) from the surface of the inclusion ($x = 0$) to the outermost layer of the interphase can be approximated as

$$P = P_{in} + (P_{out} - P_{in})\left(\frac{x}{L}\right)^n \approx P_{in} + (P_{out} - p_{in})\left(\frac{\alpha - 1}{N}\right)^n \quad (2)$$

where P represents the interphase property, n is the interphase variation parameter, with α varying in the range of $1–N + 1$.

An analytical approach [20,21] is used for modeling the effective reinforcing modulus (E_{ERM}) of a wavy CNF as

$$E_{ERM} = \frac{FL}{A\delta_F} \quad (3)$$

where F is the force acting at the free end of a clamped-free CNF, L is the length of CNF, A is the cross-sectional area of CNF, and δ_F is the deflection at the free end. Using the Castigliano's second theorem, δ_F is found using the strain energy approach as

$$\delta_f = \int \frac{Q}{EA}\frac{\partial Q}{\partial F} ds + \int \frac{kV}{GA}\frac{\partial V}{\partial F} ds + \int \frac{M}{EI}\frac{\partial M}{\partial F} ds \quad (4)$$

where Q is the internal axial force, V is the transverse shear force, k is the correction factor for the shear strain energy, M is the bending moment, I is the area moment of inertia of CNF, and ds is the differential length along the wavy CNF. The E_{ERM} is a representative value that accounts for the reduction in stiffness of a wavy CNF compared with a straight CNF of modulus E_f [20]. Hence, E_{ERM} ($E_{ERM} \leqslant E_f$) is a material parameter influenced by both the geometry and material property of the wavy CNF.

Shear-lag theory is combined with quasi-isotropic lamination (QIL) approximation [17] to calculate the strength of nano-enhanced matrix. In QIL, the mechanical properties of a ply made of enhanced matrix with aligned CNFs are calculated first. Then, the plies are stacked in a quasi-isotropic laminate (e.g. $[0/45/-45/90]_s$) to calculate the in-plane properties of an enhanced matrix with randomly oriented CNF reinforcement. Assuming the out-of-plane properties are equal to the in-plane values, QIL approximates the effect of three-dimensional random orientation of nanoreinforcements in the matrix. As shown by Garg et al. [22] and Rouhi et al. [17], this simplified model provides reasonably accurate results for very low CNF volume fractions ($V_{CNF} < 0.01$).

3. Uncertainty representation and propagation

A circular crush tube made of laminated hybrid composite material with CNF-reinforced matrix and long carbon fibers as shown in Fig. 2 is considered. Vinyl ester is selected as the base matrix material. Due to the lack of experimental data, the random material parameters are assumed to follow Gaussian distribution

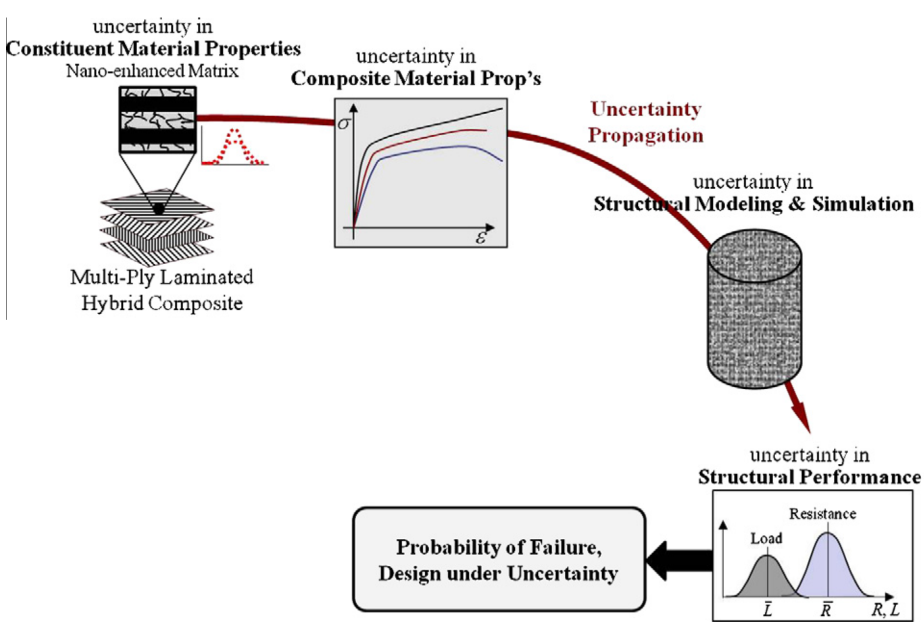

Fig. 1. Propagation of uncertainties in nano-enhanced polymer composite materials and structures.