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Review

Vibrations of straight and curved composite beams: A review

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ABSTRACT

Laminated composite straight and curved beams are frequently used in various engineering applications. This work attempts to review most of the research done in recent years (1989–2012) on the vibration analysis of composite beams. This review is conducted with emphasis given to the theory being applied (thin, thick, layerwise), methods for solving equations (finite element analysis, differential transform and others) experimental methods, smart beams (piezoelectric or shape memory), complicating effects in both material and structure (viscoelastic, rotating, tip mass and others) and other areas that have been considered in research. A simple classic and shear deformation model would be explained that can be used for beams with any laminate.

Contents

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1. Introduction

Laminated composite beams, plates and shells have been used in extensive applications of many engineering fields in recent decades. Structures composed of composite materials offer lower weight and higher strength and stiffness than those composed of most metallic materials. These advantages coupled with the ability to tailor designs for specific purposes, gave them a competitive edge when compared with normal engineering materials and led to their extensive use. Composite beams, plates and/or shell components have found increasing use in recent aerospace, submarine automotive structures.

One of the important problems in composite structural design is the vibration analysis of composite beams. Composite beams act as lightweight load carrying structures in diverse application.

Literature on composite beam research can be found in many conferences and journals. Kapania and Raciti [1] made a review on advances in analysis of laminated beams and plates vibration and wave propagation in 1989. Rosen [2] reviewed the research on static, dynamic, and stability analysis of pretwisted rods and beams in 1991. Chidamparam and Leissa [3] reviewed the published literature on the vibrations of curved bars, beams, rings and arches of arbitrary shape which lie in a plane in 1993. Also a book [4] was dedicated to vibration of composite beams, plates and shells.

This article focuses on the last two decades of research (1989–2012) done on the vibration analysis of composite beams. The literature is reviewed while focusing on various aspects of research. We will first review the various beam theories that are being used in research in recent years. These include thin (or classical), thick (or shear deformation), and layerwise beam theories. Then different methods for solving equations of motion such as transfer matrix method, finite element method and others would be reviewed. Another aspect of research will be use of smart materials, which include electrorheological fluids, piezoelectric sensors and actuators, and shape memory materials. Complicating effect will be the final category that will be addressed. This will include viscoelastic effects, added mass, rotating beam, beams with damages and so on.

2. Beam theories

Beams are generally three dimensional (3D) bodies bounded by four, relatively close surfaces. The 3D equations of elasticity are unnecessarily complicated when written for a beam. Researchers simplify such equations by making certain assumptions for particular applications. Almost all beam theories reduce the 3D elasticity problem into a one dimensional (1D) problem.

There are two issues typically treated for 1D analysis of beams. The first problem is the issue of coupling and how to include the various couplings (stretching bending, bending twisting and others) that are ignored in reducing 3D equations to 1D. A suitable approach for inclusion of coupling parameters is to redefine stiffness parameters such that it includes other couplings. Coupling problem can be solved by using equivalent stiffness parameters instead of normal definition of A_{11}, B_{11}, and D_{11}. That is why different definitions of stiffness parameters are presented first.

2.1. Stiffness parameters

Kaw [5] suggested to define the stiffness parameters based on the flexibility matrix (the ABD matrix inverse). Rios and Chan [6] proposed another formulation based on ABD inverse matrix. Ecsedi and Dluhi [7] analyzed the static and dynamic behavior of nonhomogeneous curved beams and closed rings. Instead of ABD terms, the modulus was treated as a function in cylindrical coordinates. Some other researchers [8–11] treated the problem of laminated composite beams to that of isotropic homogeneous beams with effective bending (EI), torsional (GJ), and axial stiffness parameters (EA). To the knowledge of authors, none of these models were accurate in analysis of generally laminated beams. Hajianmaleki and Qatu [12] showed that using equivalent modulus of elasticity of each lamina, one can get accurate results for static and dynamic analyses of generally laminated beams with any kind of coupling. The equivalent modulus of elasticity of each lamina is found using [12,13]

$$\frac{1}{E_x^k} = \frac{\cos^4(\theta_k)}{E_{11}} + \left(\frac{1}{G_{12}} - \frac{2\upsilon_{12}}{E_{11}}\right)\cos^2(\theta_k)\sin^2(\theta_k) + \frac{\sin^4(\theta_k)}{E_{22}} \tag{1}$$

Equivalent A_{11}, $B_{11}$and D_{11} using the following equations were suggested for a curved beam [14]

$$\begin{aligned}
A_{11} &= R\sum_{k=1}^{N} bE_x^{(k)} \ln\left(\frac{R+z_k}{R+z_{k-1}}\right)\\
B_{11} &= R\sum_{k=1}^{N} bE_x^{(k)} \left[(z_k - z_{k-1}) - R\ln\left(\frac{R+z_k}{R+z_{k-1}}\right)\right]\\
D_{11} &= R\sum_{k=1}^{N} bE_x^{(k)} \left[\frac{1}{2}\left((R+z_k)^2 - (R+z_{k-1})^2\right) - 2R(z_k - z_{k-1})\right.\\
&\quad \left. + R^2 \ln\left(\frac{R+z_k}{R+z_{k-1}}\right)\right]
\end{aligned} \tag{2}$$

and for a straight beam, they reduce to [15,16]

$$\begin{aligned}
A_{11} &= \sum_{k=1}^{N} bE_x^k (h_k - h_{k-1})\\
B_{11} &= \sum_{k=1}^{N} bE_x^k \frac{(h_k^2 - h_{k-1}^2)}{2}\\
D_{11} &= \sum_{k=1}^{N} bE_x^k \frac{(h_k^3 - h_{k-1}^3)}{3}
\end{aligned} \tag{3}$$

2.2. Effect of shear deformation

The inclusion of shear deformation in the analysis of beams was first made by Timoshenko [17]. Hence, theories considering shear deformation are called as Timoshenko beam theories. In this regard, the beam theories are classified based on the order of polynomial for approximation of displacements through the thickness. Fig. 1 shows a free body diagram of a differential beam element. Suppose that the displacement u can be expressed as