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An exact analytical approach for free vibration of Mindlin rectangular nano-plates via nonlocal elasticity

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ABSTRACT

Eringen nonlocal theory is employed in Mindlin plate theory to consider small scale effects on free vibration of rectangular nano-plates. Introducing some auxiliary and potential functions, an exact analytical procedure is applied on the governing equations to decouple the displacement variables. It is believed that this method is new for solving vibration of nano-plates. The solution of natural frequencies is obtained for Levy-type boundary conditions (two opposite edges simply supported and the others arbitrary). In order to confirm the reliability of the method considered, the results are compared with several reported literature. The effect of nonlocal parameter is investigated on natural frequency of the nano-plate for different boundary conditions. Finally the influence of aspect ratio and thickness to length ratio on natural frequency is studied in detail. It is expected that results obtained in this paper serve as an accurate reference in future nano-structures issues.

1. Introduction

Among different nano-structures, one can introduce nano-plates which concern with small scale fields. Due to the rapid development of technology, especially in micro and nano-scale fields, nano-plates are used in micro- or nano-electromechanical systems (MEMS or NEMS) for their superior mechanical, thermal and electrical properties. Dynamic behavior of nano-plates used as thin film elements [1], two-dimensional suspended nano-structures [2,3] nano-sheet and paddle-like resonators [4,5] requires a two-dimensional nano-structure analysis. Hence, one must consider small scale effects in order to refine classical theories to derive the governing equations for these structures. The scale effects are accounted by considering internal size as a material parameter. Experimental results show that as length scales of a material are reduced, the influences of long-range interatomic and intermolecular cohesive forces on the mechanical properties become prominent and cannot be neglected. The local (classic) continuum theory neglects the effects of long-range load on the motion of the body and long range inter atomic interactions. Therefore, the internal scale is neglected. Some methods like molecular dynamics [6] are presented in recent years which consider size effects and atomic lengths. Molecular dynamics models are limited to the small number of atoms and relatively short times. Therefore, the simulation time (cost) increases enormously if we increase the length and the number of atoms. Nonlocal linear theory, which has both features of lattice parameter and classical elasticity, could be considered a superior theory for modeling nano-materials. Nonlocal theory of Eringen [7] is one of the well-known continuum mechanics theories to account the small scale effect by specifying the stress at a reference point as a functional of the strain field at every point in the body. Hence, many papers dealt with analyzing nano-structures have been published on this topic. Buckling and vibration analyses of carbon nano-tubes with the help of beam and shell theories [8,9], application of nonlocal theory for beam vibration [10] and vibration analysis of graphite sheets using the plate theories [11] are some of the wide application of nonlocal theory. Study of the vibration and buckling analysis of nano-plates and graphene sheets can be seen in bending and vibration of plates via nonlocal Reddy plate theory [12], CPT and Mindlin nonlocal theory for plate vibration [13,14], free vibrations of single-layered graphene sheets [15], buckling of graphene sheets [16,17], vibration and buckling of nano-plates [18] and 3D vibration analysis of nano-plates [19]. But as reported in many of these literature the solution of the governing equation are based on numerical methods (e.g., finite element method [20], finite difference method [21], differential quadrature method [22]) and approximate analytical methods like Navier type solution method that assumes the variation of displacement variables harmonically [12,13]. Furthermore, many of these solutions are concerned with Navier boundary condition, i.e. all edges are simply supported and a few of them consider combinations of clamed and simply supported boundaries [18,22]. Hence, no exact closed-form

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Nomenclature

a, b	plate length and width	u_i	components of displacement
t	time	u_r^0	mid-plane displacements in x- and y-directions
l	internal length	φ_r	rotational displacements about the x- and y-axes
e_0	material constant	$\tilde{\varphi}_r$	non-dimensional displacements about the x- and y-axes
ρ	mass density	w	lateral displacement in z-direction
h	nano-plate thickness	$\tilde{w}$	non-dimensional lateral displacement in z-direction
E	Young modulus of elasticity	κ, μ	nonlocal parameters
ν	Poisson's ratio	ζ	non-dimensional nonlocal parameter
D	flexural rigidity	nl	nonlocal term
x, y, z	rectangular Cartesian coordinates	k_s	shear correction factor
X, Y	non-dimensional rectangular Cartesian coordinates	I_k	inertia terms
ε_{ij}	strain components	δ	thickness to length ratio
t_{ij}	nonlocal stress tensor	η	aspect ratio (length to width ratio)
σ_{ij}	local stress tensor	β	non-dimensional frequency parameter
N_{ij}	nonlocal force resultants	ω	frequency parameter
M_{ij}	nonlocal moment resultants	$\alpha(\lvert x'-x\rvert)$	nonlocal kernel function
p_i	components of body force	$f(X,Y)$	auxiliary function
$\mathcal{L}$	linear differential operator	$W_t(X,Y)$	potential functions
∇^2	two-dimensional Laplacian operator	m, n	number of half waves in x- and y-directions
$\tilde{\nabla}^2$	non-dimensional Laplacian operator		

solution is available in the literature for the free vibration analysis of nano-plates and various Boundary conditions (BCs). According to Hosseini-Hashemi et al. [23,24] an exact closed form solution procedure is established for vibration of single-layered and functionally graded plates based on some auxiliary and potential functions. This method has just been considered for local theory and can be applied to Levy-type support conditions and yield well convergence and accurate results without any approximations. Therefore, the main purpose of this article is to apply this exact method to solve the governing equations of motion of nano-plate for Mindlin theory base on nonlocal elasticity. In this regard, the rectangular plate equations of motion for Mindlin theory are derived via equations of momentum balance and base on nonlocal continuum model. The equations of the problem are coupled through displacement components. Introducing a set of auxiliary and potential functions, the governing equations are decoupled for transverse vibration analysis. By transforming the displacement variables into known functions the problem leads to a soluble form without any approximations. Two opposite edges are held simply supported and the other two edges may be given any combination of free (F), simply supported (S) and clamped (C). Applying the boundary conditions lead to characteristic equations which result natural frequencies accurately and analytically. In order to confirm the reliability of the method considered, the results are compared with several reported literature. Also, the effects of nonlocal parameter, aspect ratio and thickness to length ratio of the plate and different boundary conditions on non-dimensional vibration frequencies are investigated.

2. Problem formulation

2.1. Summary of nonlocal continuum theory

As mentioned earlier in nonlocal theory the stress in a material body point is a function of strain field of the same point and all other ones in material domain, so the stress tensor plays the essential role in this continuum theory which is defined as [7]:

$$t_{ij} = \int_V \alpha(|x'-x|)\sigma_{ij}(x')dV' \tag{1}$$

where the volume integral is taken over the body region V. x is the reference point in body which the stress tensor is calculated at, x' any other point in the body, $i, j = x, y, z$, for three dimensional Cartesian coordinate, σ_{ij} is the local stress tensor and $\alpha(|x'-x|)$ is nonlocal kernel function depends on internal characteristic length. Eringen proposed $\alpha(|x'-x|)$ as a Green function of a linear differential operator $\mathcal{L}$ as:

$$\mathcal{L}\alpha(|x'-x|) = \delta(|x'-x|) \tag{2}$$

Which after applying Eq. (2) on Eq. (1) the integral forms of nonlocal stress tensor reduces to differential one:

$$\mathcal{L}t_{ij} = \sigma_{ij} \tag{3}$$

The linear operator is an approximate model of the kernel obtained by matching the Fourier transforms of the kernel in the wave number space with the dispersion curves of lattice dynamics. For curve-fitting at low wave numbers relevant to the small internal length scale Eq. (2) is written as:

$$(1-\kappa^2\nabla^2+\mu^4\nabla^4-\ldots)t_{ij} = \sigma_{ij}$$

So the linear operator becomes:

$$\mathcal{L} = (1-\kappa^2\nabla^2+\mu^4\nabla^4-\ldots) \tag{4}$$

where κ and μ are small parameters proportional to the internal length scale. If first order approximation is to be considered, just the Laplacian form of the operator in Eq. (4) is maintained [27]. So for the two-dimensional case:

$$\mathcal{L} = 1-(e_0 l)^2\nabla^2 \tag{5}$$

In which l is internal length and e_0 is material constant which is defined by the experiment and $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplacian operator.

Equations of motion for nonlocal linear elastic solids are obtained from nonlocal balance law as:

$$t_{ij,j}+p_i = \rho\ddot{u}_i \tag{6}$$

p_i and u_i are the components of the body force and displacement vector respectively and ρ is mass density. Using Eq. (3) in Eq. (6) the nonlocal equations of motion in differential form become:

$$\sigma_{ij,j}+\mathcal{L}(p_i-\ddot{u}_i) = 0 \tag{7}$$

It should be noted that the boundary conditions here are based on nonlocal stress tensors t_{ij} rather than local ones σ_{ij} [13].