Composite Structures 100 (2013) 373–384

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Coupled free vibration of composite beams with asymmetric cross-sections

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ARTICLE INFO

Article history:
Available online 21 January 2013

Keywords:
Coupled free vibration
Composite beams
Asymmetric cross-sections
Euler–Bernoulli beam theory

ABSTRACT

A set of linear differential equations of motion for coupled free vibrations of composite beams with asymmetric cross-sections is obtained in this paper based on Euler–Bernoulli beam theory. More coupling terms are included in the governing equations; and hence, both even and odd order spatial derivatives are included. All these features lead to much difficulty in solving the resultant equations. The axially unloaded condition of $F_1 = 0$, instead of $e = 0$, is adopted to simplify the equations of motion for the case of inextensional beams. An algorithm is developed for solving the resultant equations to obtain natural frequencies and modes of the beams. Numerical examples for validation show that the equations of motion obtained in this paper are correct and the corresponding algorithm is effective and easy to use.

1. Introduction

Composite beams are becoming common structures in modern engineering, such as wind turbine blades and high aspect ratio wings. A feature of these structures is that their material properties are anisotropic and cross sections are asymmetric which means that the elastic center and mass center of cross sections do not coincide with each other so that Flexural–Flexural–Torsional (F–F–T) coupled vibrations occur. This increases difficulty in vibration analysis of these structures. Besides, if the amplitudes of vibrations are significant enough, geometric nonlinearities need to be considered.

In nonlinear F–F–T coupled vibration analysis of beam structures, Crispo Der Silver and Glynn [1] first developed a set of differential equations of motion for general isotropic beams based on three Euler angles describing the motion of the beams. Pai and Nayfeh [2] generalized Crispo Der Silver and Glynn's equations of motion to composite beams. In their researches [1,2], the equations of motion were reduced for the case of inextensional beams based on the condition of $e = 0$, where $e = u' + \ldots$, is the axial strain of the reference line of the beams. This implies that the elastic center and mass center of each cross section of the beams coincide with each other; or, the cross sections are bi-symmetric. Beran, et al. [3] developed a set of nonlinear equations of motion for isotropic beams with one symmetric axis in each cross section, with the reference lines coincident with the elastic centers of the beams.

The analysis of nonlinear vibrations is closely associated with the corresponding linear vibration analysis. For example, in perturbation method, the method of multiple scales, etc., linear modes are often used for spatial discretization. In conventional nonlinear vibration analysis, pure flexural and torsional natural modes are used even in flexural–torsional coupled cases [4,5]. In some linear vibration analysis, such as first order flutter analysis of high aspect ratio wings, pure flexural and torsional natural modes are also used in flexural–torsional coupled cases [5].

The existing solution methods of linear equations of motion for flexural–torsional coupled vibrations can be divided into two categories. In the first category, the high order differential equations are solved directly [6–14]. Many arbitrary constants need to be dealt with through a complicated procedure. In order to reduce the effort in the solution procedures, modern computer software, such as Mathematica, has been adopted. For example, Tanaka et al. [10] solved the final equations for natural frequencies with the help of Mathematica. In the second category, the original high order differential equations are transformed into a set of first order differential equations and solved by means of conventional methods for first order ordinary differential equations (ODEs) [15,16].

In both categories above, the reference lines of the beams are coincident with elastic axes. For the determination of natural frequencies, the most adopted general algorithm is the dynamic stiffness method given by Wittrick and Williams [17], wherein natural frequencies are solved in real number domain. In these solutions, the governing differential equations of motion include only even order spatial derivatives, such as x'' and x'''', and the eigenvalues can be determined qualitatively as real numbers or pure imaginary numbers. Therefore, the solution procedures can be conducted in real number domain and the algorithm given by Wittrick and Williams can be adopted without difficulty. However, if the differential equations of motion include both even and odd order spatial derivatives, such as x', x'', and x''', the eigenvalues cannot be determined qualitatively as real numbers or pure imaginary numbers and the conventional dynamic stiffness method may not be applied directly. It seems that a general algorithm is needed to solve such a problem in the complex domain.

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http://dx.doi.org/10.1016/j.compstruct.2012.12.054

In this paper, the nonlinear differential equations of motion given by Nayfeh and Pai [18] are extended to the case of F–F–T coupled vibrations of composite beams with asymmetric cross sections; and then, a set of general linear differential equations of motion are obtained for analyzing natural vibrations of such beams, based on Euler–Bernoulli beam theory. The axial unloaded condition of $F_1 = 0$, instead of $e = 0$, is used to simplify the equations of motion for the case of inextensional beams, where F_1 is the resultant of axial stress σ_{11} in the cross sections of the beams. The resultant equations include both even and odd order spatial derivatives. A unified algorithm is developed to obtain natural frequencies and modes of the beams. The corresponding solution procedure is conducted with the help of Mathematica.

In order to validate the correctness of the governing equations and the effectiveness of the algorithm developed in this paper, three numerical examples are presented. The first one is a typical undamped free vibration problem of a flexural–torsional-uncoupled isotropic cantilever beam, whose exact solutions are available. This example is used to validate the correctness of the main feature of the present governing equations and corresponding algorithm. The second one is the numerical example of a composite beam presented by Pai [16]. This example is used to validate the effectiveness of the present algorithm in composite beams. The last one is a numerical example given by Tanaka et al. [10]. This example is used to validate the effectiveness of the present algorithm in beams with asymmetric cross sections. Besides, in order to compare with the Tanaka's example which includes warping effect, the equations of motion of this paper are also extended to include warping effect as those in [10].

2. Equations of motion

2.1. Fundamental formulation

A cantilever beam is taken as an example in deriving the equations of motion of this paper. The results obtained in this paper can be easily extended to beams with other boundary conditions. Warping effect due to torsional vibration is discussed only for the application in the last numerical example for validation.

An initially straight beam presented in [18] is adopted in this paper, as shown in Fig. 1a. X–Y–Z is a fixed global Cartesian coordinate system to describe the undeformed geometry of the beam. ξ–η–ς is an orthogonal curvilinear coordinate system located in each cross section of the beam. In the undeformed state, ξ-axis coincides with X-axis and they all coincide with the reference line that passes through the elastic center of each cross section of the beam, η-axis coincides with Y-axis, etc.

An arbitrary cross section of the beam is shown in Fig. 1b where e_η and e_ς are the distances between elastic center (S) and mass center (C) in the cross section along η, ς directions respectively. The beam is assumed to be an Euler–Bernoulli beam. If the warping effect is neglected, the cross sections remain planes after deformation. $u(s)$, $v(s)$, $w(s)$ are used to describe the displacements of the reference point (that is, the elastic center) in the cross section along X, Y, Z directions respectively, where s is the undeformed length measured from the clamped end of the beam to the reference point of the cross section. $\theta_\xi(s)$, $\theta_\eta(s)$, $\theta_\varsigma(s)$ are used to describe the rotations of the cross section about ξ, η, ς axes respectively. Only u, v, w, θ_ξ are required in deriving the equations of motion since θ_η, θ_ς can be expressed as functions of u, v, w[18]. In this paper, $\theta_\xi(s)$ is denoted by $\phi(s)$.

In order to obtain the differential equations of motion, the fundamental nonlinear Eqs. (4.6.28–4.6.31) and the corresponding boundary conditions (4.6.32a–c) in [18] are adopted in this paper. By locating the reference line with the elastic centers of the beam coincidently and carrying out Talyor-series expansion based on the transformation with two Euler angles in the fundamental nonlinear equations given in [18] and dropping off all nonlinear terms, damping terms and external forces in the resultant nonlinear equations, the following 3-D linear differential equations of motion for natural vibration analysis are obtained

$$m\ddot{u} - J_{12}\ddot{w}' - J_{13}\ddot{v}' - A_{11}u'' - B_{11}\phi'' + B_{12}w''' - B_{13}v''' = 0 \quad (1a)$$
$$m\ddot{v} - J_{12}\ddot{\phi} + J_{13}\ddot{u}' - j_{23}\ddot{w}'' - j_{33}\ddot{v}'' + B_{13}u''' + D_{13}\phi''' - D_{23}w^{(4)} + D_{33}v^{(4)} = 0 \quad (1b)$$
$$m\ddot{w} + J_{12}\ddot{u}' + J_{13}\ddot{\phi} - j_{22}\ddot{w}'' - j_{23}\ddot{v}'' - B_{12}u''' - D_{12}\phi''' + D_{22}w^{(4)} - D_{23}v^{(4)} = 0 \quad (1c)$$
$$j_{11}\ddot{\phi} - J_{12}\ddot{v} + J_{13}\ddot{w} - B_{11}u'' - D_{11}\phi'' + D_{12}w''' - D_{13}v''' = 0 \quad (1d)$$

where the overdot '·' denotes the derivative with respect to t and the prime ''' denotes the derivative with respect to s. The coefficients in Eq. (1) are listed in Appendix A.1. It can be seen that more inertial and elastic coupling terms are included in Eq. (1), and both even and odd order spatial derivatives occur in it.

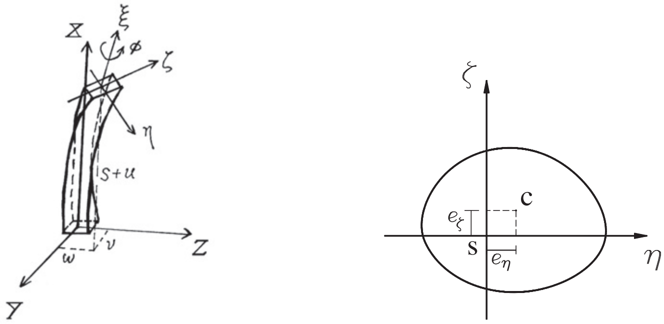
For cantilever beams, the boundary conditions corresponding to Eq. (1) can be given as follows [18].

At the clamped end ($s = 0$),

$$u = v = w = \phi = 0 \quad (2a)$$
$$v' = w' = 0 \quad (2b)$$

At the free end ($s = L$, where L is the length of the beam),

$$M_1 = B_{11}u' + D_{11}\phi' - D_{12}w'' + D_{13}v'' = 0 \quad (3a)$$
$$M_2 = B_{12}u' + D_{12}\phi' - D_{22}w'' + D_{23}v'' = 0 \quad (3b)$$
$$M_3 = B_{13}u' + D_{13}\phi' - D_{23}w'' + D_{33}v'' = 0 \quad (3c)$$
$$G_1 = A_{11}u' + B_{11}\phi' - B_{12}w'' + B_{13}v'' = 0 \quad (3d)$$
$$G_2 = -B_{13}u'' - D_{13}\phi'' + D_{23}w''' - D_{33}v''' + j_{23}\ddot{w}' + j_{33}\ddot{v}' - J_{13}\ddot{u} = 0 \quad (3e)$$
$$G_3 = B_{12}u'' + D_{12}\phi'' - D_{22}w''' + D_{23}v''' + j_{22}\ddot{w}' + j_{23}\ddot{v}' - J_{12}\ddot{u} = 0 \quad (3f)$$



(a) An initially straight beam [18] (b) An arbitrary asymmetric cross section

Fig. 1. A cantilever beam with arbitrary bi-asymmetric cross sections.