Composite Structures 100 (2013) 511–516

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Theoretical prediction on the mechanical properties of 3D braided composites using a helix geometry model

Lili Jiang [a], Tao Zeng [a,*], Shi Yan [a], Daining Fang [b]

[a] Department of Engineering Mechanics, Harbin University of Science and Technology, Harbin 150080, PR China
[b] Department of Mechanics and Aerospace Technology, Peking University, 100871 Beijing, PR China

ARTICLE INFO

Article history:
Available online 29 January 2013

Keywords:
Helix geometry model
3D braided composites
Effective elastic properties
Strength

ABSTRACT

In our previous work, we have established a three-dimensional (3D) finite element model (FEM) which precisely simulated the spatial configuration of the braiding yarns. This paper presents a theoretical model based upon the helix geometry unit cell for prediction of the effective elastic constants and the failure strength of 3D braided composites under uniaxial load through the stiffness volume average method and Tsai-Wu polynomial failure criterion. Comparisons between the theoretical and experimental results are conducted. The theoretical results show that the braid angle has significant influences on the mechanical properties of 3D braided composites.

1. Introduction

3D braided composites have been rapidly developed in the past years due to their excellent mechanical properties, such as high specific strength/stiffness, high through-thickness strength and impact resistance. 3D braided composites have been widely used in aerospace, automobile, marine and biomedical, etc. 3D braided composites can be regarded as an assemblage of representative volume element [1,2] that captures the major features of the underlying microstructure and composition in the material. In the recent years, many researchers [3–22] have been devoted to the micro-structures and elastic properties for 3D braided composites. Kregers and Melbardis [3] presented the stiffness volume average method to predict the macroscopic properties of 3D braided composites. Ma et al. [4], Yang et al. [5] and Byun et al. [6] studied the effective elastic properties of 3D braided composites by using ‘Fiber interlock model’, ‘Fiber inclination model’ and ‘fabric geometric model’, respectively. Whitcomb and Woo [7] gave the stress distribution of woven composites using the local finite element method. Wang and Wang [8] reported a mixed volume averaging technique to predict the mechanical behavior of three dimensional braided composites. Wu [9] developed a three-cell model to predict the mechanical properties of 3D braided composites, which can be used to accurately describe the micro-structure. Chen et al. [10] presented a finite multi-phase element model to predict the effective properties of 3D braided composites. Sun and Qiao [11] predicted the strength of 3D braided composites based upon the transverse isotropy of unidirectional laminas. Fang et al. [12] developed a mesoscopic damage model to study the failure locus of 3D braided four-directional composites under complex loadings. Zeng et al. [13,14] investigated the effective modulus of 3D braided composites with edge and internal cracks. Gu [15] investigated the uniaxial tensile strength of 4-step 3-dimensional braided composites based on the energy conservation, and showed the tensile curve within the whole strain range. Yu and Cui [16] studied the influence of the braiding angle and the fiber volume fraction on the strength for 4-step braided composites. Li and Shen [17] presented thermal postbuckling analysis modeling for 3D braided composite cylindrical shell subjected to a uniform temperature rise. Recently, the finite element methods [18–23] were extensively applied to numerically predict the average stiffness and strength properties of 3D braided composites for their accurate prediction. In our previous work [24], a multiphase finite element method based on the helix geometry model has been presented to predict the effective elastic constants and strength of 3D braided composites under tension loading.

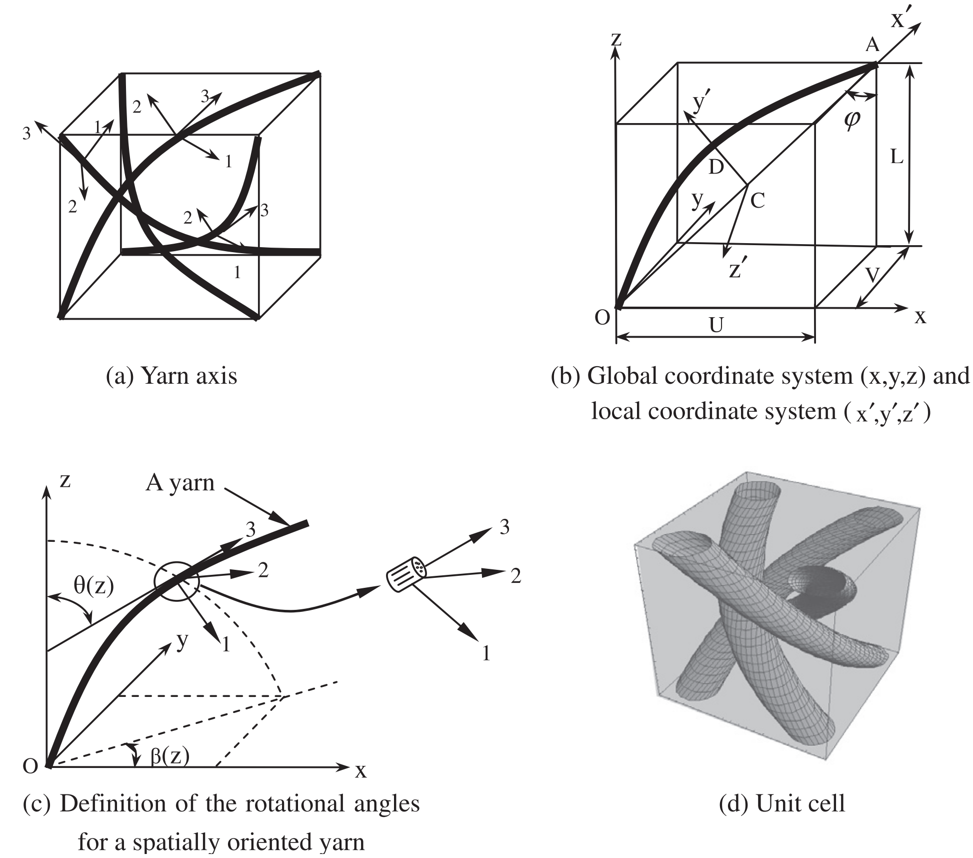
The present paper is concerned with the theoretical prediction on the elastic properties and failure strength of 3D braided composites using a helix geometry model. The stiffness property is first compared with test data and the results of the previous micromechanical models. This study is followed by predicting the failure strength of 3D braided composites under axial load.

2. Helix geometry model

In our previous work [24], a helix geometry model of 3D braided composites has been presented. A unit cell for the helix geometry model of 3D braided composites is shown in Fig. 1. Four yarns in the helix geometry model are curved to avoid the collision at the

* Corresponding author. Tel.: +86 451 86390832; fax: +86 451 86390830.
E-mail addresses: taozeng@sohu.com, taozeng@hrbust.edu.cn (T. Zeng).

0263-8223/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2013.01.016



(a) Yarn axis

(b) Global coordinate system (x,y,z) and local coordinate system (x′,y′,z′)

(c) Definition of the rotational angles for a spatially oriented yarn

(d) Unit cell

Fig. 1. Helix geometry model.

center of the unit-cell, which truly reflects the braided manner and coincides with the actual configuration of the braided composites. Three coordinate systems have been employed in this study, which are: (1) a global coordinate system (x, y, z) with axes aligned parallel to the unit cell axes, (2) a local coordinate system (x', y', z') with its origin C at the midpoint of the diagonal of the unit cell (x'-axis: the diagonal direction of unit cell); (3) a local coordinate system (1, 2, 3) with its primary axis parallels to the central axis of the yarn. The local system (1, 2, 3) changes from point to point on a yarn as well as from yarn to yarn. U, V and L refer to the dimensions of the unit cell in x, y and z directions. In order to describe the spatial location of the yarns in the unit cell, the curvature of each yarn should be determined. The center line of the braiding yarns is a parabola defined by the two yarn end points (located on the respective top and bottom surfaces of the unit cell) and the midpoint of the yarn (located on the mid-plane between the top and bottom surfaces), as shown in Fig. 1b.

The center line of the braiding yarns in the local coordinate systems (x', y', z') can be formulated as

$$\begin{cases} y' = c_1 + c_2 x' + c_3 x'^2 \\ z' = 0 \end{cases} \tag{1}$$

where c_1, c_2, c_3 can be determined by the two end points and the midpoint of the yarn.

The center line of the braiding yarns in the global coordinate systems (x, y, z) can be obtained by coordinate transformation

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \begin{Bmatrix} x - \frac{U}{2} \\ y - \frac{V}{2} \\ z - \frac{L}{2} \end{Bmatrix} \tag{2}$$

where (l_i, m_i, n_i) $(i = 1, 2, 3)$ are the direction cosines between the local coordinate system (x', y', z') and the global coordinate system (x, y, z).

Substituting Eq. (2) into Eq. (1), the equation of the center line of the braiding yarns in the global coordinate system can be obtained as

$$\begin{cases} x = f_1(z) \\ y = f_2(z) \end{cases} \quad 0 \leqslant z \leqslant L \tag{3}$$

The expressions of $f_1(z)$ and $\theta(z)$ are listed in Appendix A.

In Fig. 1c, $\theta(z)$ is the angle between the tangent of the yarn axis and z-axis and $\beta(z)$ is the angle between the projection of yarn axis on the xoy plane and x-axis, given by:

$$\theta(z) = \arccos \frac{1}{\sqrt{\left(\frac{dx}{dz}\right)^2 + \left(\frac{dy}{dz}\right)^2 + 1}} \tag{4a}$$

$$\beta(z) = \arctan\left(\frac{dy}{dx}\right) \tag{4b}$$

3. Prediction of effective elastic constants

The basic assumption in the present analysis is that the yarns (Fig. 1d) are considered unidirectional composite rods after resin impregnation. A yarn was cut into a few small pieces along the braiding axis z so that the yarns segment in each small piece was assumed to be straight. Each small piece was treated as a transversely isotropic composite with local coordinate system along its yarn segment. The stiffness matrix of each yarn segment in