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Postbuckling optimisation of variable angle tow composite plates

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ABSTRACT

The potential for enhanced postbuckling performance of flat plates using variable angle tow (VAT), in comparison with conventional laminated composites, has been shown previously. This paper presents an optimisation strategy for the design of postbuckling behaviour of VAT composite laminates under axial compression. The postbuckling performance of composite laminated plates for a given compression loading is assessed by studying both the maximum transverse displacement and the end-shortening strain. For the postbuckling analysis of VAT composite plates, an efficient tool based on the variational principle and the Rayleigh–Ritz method is developed. In the optimisation study, a mathematical definition based on Lagrangian polynomials, which requires few design parameters, is used to define a general fiber angle distribution of the VAT plate. A genetic algorithm is subsequently used to determine the optimal VAT configuration for maximum postbuckling performance. The optimisation of square VAT laminates under compression loading for different in-plane boundary conditions is studied and compared with straight fiber designs.

1. Introduction

Use of laminated composite structures in primary load carrying aircraft structures has increased substantially over the past decade (with, for example, Airbus A380 and Boeing 787). The advent of new commercial composite manufacturing techniques such as advanced fiber placement and advanced tow placement, as well as emerging constant tow shearing [1], enable steering of tow paths in the plane of the structure and allow manufacturing of composite structures with variable stiffness. These laminates, with spatially varying fiber (tow) orientations, i.e. variable angle tow (VAT) composites, allow point-wise stiffness tailoring and have been shown to exhibit numerous structural advantages over straight fiber composites. Previous research has shown great potential of utilising VAT concept for improving structural performance, particularly the buckling and postbuckling load carrying capability of composite plates [2–6]. Despite the benefits of VAT laminates, the optimum design of such laminates is a challenge, as it involves a large number of design variables and large computational effort required by an optimisation process. These challenges in the design of VAT composite plates necessitate the development of new efficient modelling and effective optimisation strategies. Several optimisation works on the design of VAT plates for maximum buckling load have been proposed, which are based on either using genetic algorithms (GA) [3,7] or gradient-based mathematical programming techniques [8,9]. In this work, numerical studies using GA for the postbuckling design of VAT laminates under axial compression, are presented.

For the design of VAT laminates, it is both desirable and necessary to systematically represent general laminate configurations, as it reduces the number of design variables considerably. VAT laminates can be described using mathematical functions to define either the tow (fiber) trajectories, or the variation of fiber orientation angles. In the literature, existing schemes for the representation of VAT laminates are categorised into two classes. The first description is coupled with finite-element modelling, in which the design parameter in each element (node) is assumed to be constant and is allowed to independently vary across elements. For this element-based scheme, however, an additional constraint is often needed to ensure the variation of design parameters to be smooth [10].

Hyer and Lee [11] first demonstrated the use of curvilinear fibers to enhance buckling loads for a plate with a circular hole by optimising fiber orientations of each element. Huang and Haftka [12] used a hybrid strategy to optimise fiber angles at each node in their finite element model for improving the strength of a laminate with a hole. More recently, Abdalla et al. [10], Setoodeh et al. [8] and IJsselmuiden et al. [9] performed the optimum design of tow-steered variable stiffness composite laminates based on weighted nodal design variables. Another way of representing VAT configurations is to employ a mathematical definition, which requires fewer design variables than the element discretization scheme and can naturally generate smooth distributions. For

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example, Nagendra et al. [13] proposed an optimisation strategy using NURBS (Non-uniform rational B-spline) curves with a fixed number of control points to construct the curvilinear fiber paths. Honda et al. [14] defined the fiber paths using a linear combination of B-spline functions to define the fiber trajectories. Parnas et al. expressed the fiber angles in terms of either Bezier curves or cubic polynomials [15]. Olmedo and Gürdal [2] proposed a simple definition of linear variation of fiber angels using three parameters to characterise each VAT ply. The linear variation definition has been widely used for the study [16,4], test [3] and design [3] of VAT laminates. The linear fiber angle variation along a particular direction results in a limited design space, which can be further expanded by allowing the fiber orientation to vary two dimensionally [7]. The distribution of fiber angles is then expressed in a series form, for example Setoodeh et al. [17] and Alhajahmad et al. [18] employed Lobatto polynomials. A more general definition for the nonlinear variation of fiber angles using Lagrangian polynomials [7] is used in this work to establish each VAT configuration. This definition is more intuitive for the designer as the coefficient of Lagrangian polynomials is directly equal to the fiber angle at each pre-selected control point. Furthermore, the optimisation results for the maximum buckling load in [7] also demonstrated that this nonlinear definition gives similar levels of fidelity and design space as finite element based models [9], yet has significantly reduced degrees of freedom.

Ghiasi et al. presented a thorough review and comparison of the optimisation techniques for both constant stiffness [19] and variable stiffness [20] composite laminates. Among these techniques, the genetic algorithm (GA) is one of the most popular methods which has been extensively used in the optimisation of composite laminates. Postbuckling models, either based on analytical solutions or finite element analysis (FEA), can be directly integrated into a GA to perform optimisation studies, as the gradient information is not required. The GA is generally able to prevent the search procedure becoming trapped in local optima, provided the population size and the number of generations are sufficiently large. However, GAs also suffer from significant computational costs due to numerous evaluations of the fitness function. Kang and Kim [21] combined a GA, along with a nonlinear finite element code, to perform minimum weight optimisation for composite panels subject to buckling and postbuckling design criteria. Due to the relatively poor efficiency of FEA, they implemented a parallel computing scheme with the GA to speed up the optimisation procedure. Seresta et al.[22] used GA to optimise compression loaded laminated composite plates for maximum failure load in the postbuckling regime, in which the postbuckling analysis is performed by the Rayleigh–Ritz method in a semi-analytical way. More recently, Falzon and Faggiani [23] implemented an optimisation procedure based on a GA, together with FEA (Abaqus), to optimise the layups of stiffened composite panels, for enhancing the damage resistance, in the postbuckling regime.

The main difficulty for the postbuckling optimisation is the large computational effort involved in the process of tracing the postbuckling equilibrium path, particularly when finite element modelling is used. In the postbuckling optimisation of laminated composite plates, it is often needed to determine the postbuckling behaviour of a large number of lamination configurations. Therefore, most previous works exploit analytical/semi-analytical formulations, or other approximate schemes that rapidly determine the nonlinear structural responses, so as to incorporate optimisation procedures. Dickson and Biggers [24] developed a program (POSTOP) to perform the optimum design of metal or composite panels with open-section stiffeners under combined loadings. In their program, analytical/semi-analytical solutions were adopted for the buckling and postbuckling analysis and a modified feasible-direction method was applied for the optimisation procedure. Shin et al. [25] presented a work on the minimum-weight optimum design of specially orthotropic laminates with respect to the postbuckling performance. The postbuckling analysis of composite laminates was performed using a Marguerre-type energy method and the phenomenon of mode jumping is considered in the optimisation process. Bushnell [26] also implemented the function for the postbuckling design in his PANDA2 program, in which the postbuckling problem was solved in a semi-analytical way. In addition, an optimisation strategy was introduced to retreat or remove the cases with non-convergent solutions. Other routines/tools integrated with efficient modellings were also developed and used for the postbuckling optimum design, such as NLPANOPT [27,28], VICONOPT [29,30]. As an alternative to analytical solutions, the large computational costs are reduced in some works by the application of response surface methodologies that offer a global approximation scheme to predict the postbuckling behaviour using a few number of sampled points [31,32]. In this paper, we have adopted a semi-analytical model derived from the mixed variational principle [6] to analyse the postbuckling behaviour of VAT plates. This model, is found to be more accurate and requires relatively less computational time, when compared with other postbuckling modelling techniques [5] and so enables us to perform the postbuckling optimisation using a genetic algorithm.

The aim of this paper is to present an optimisation strategy using a GA for the design of postbuckling behaviour of VAT laminated plates and study the mechanics that are responsible for the improvement of postbuckling strength. The VAT representation, using Lagrangian polynomials to define the nonlinear distribution of fiber orientation angles across the VAT plates, is introduced in the next section. Section 3 briefly reviews the proposed semi-analytical model for the postbuckling analysis of VAT plates. In Section 4, the postbuckling design criteria for the optimal design of VAT plates are first presented, followed by a detailed discussion of the optimisation strategy. The optimally designed VAT laminates are illustrated and discussed in Section 5, compared with optimal straight-fiber laminates.

2. Variable angle tow plates

In general, the distribution function that describes fiber orientation angles over a plate domain can be expanded using a double series form. A previously proposed scheme [7] that uses Lagrangian polynomials to define the nonlinear variation of fiber angles is adopted in this work for the optimal design of VAT laminates. In this definition, a set of $M \times N$ control points are first selected over the domain of the plate, each of which is associated with a value of fiber angle. A nonlinear distribution for the fiber orientation angles is generated using Lagrangian polynomials to interpolate the fiber angles at these control points, given by,

$$\theta(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} T_{mn} \cdot \prod_{m \neq i}\left(\frac{x - x_i}{x_m - x_i}\right) \cdot \prod_{n \neq j}\left(\frac{y - y_j}{y_n - y_j}\right) \tag{1}$$

where T_{mn} is the coefficient of each term in the series and is directly equal to the value of fiber angle at the control point (x_m, y_n). Fig. 1 illustrates two types of nonlinear variation of fiber orientation angles, one is parabolically varying along one direction and the other is varying two dimensionally. Substituting the coordinates of the selected control points into Eq. (1), explicit forms for describing the distribution of fiber orientation angles are formed. For example, the parabolic variation of fiber orientations along the x direction, as shown in the left plot of Fig. 1, is written as [33],

$$\begin{aligned}\theta(\xi) &= \theta(|\xi|)\\ &= T_0(2\xi^2 - 3|\xi| + 1) - 4T_1(\xi^2 - |\xi|) + T_2(2\xi^2 - |\xi|)\end{aligned} \tag{2}$$