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Composite laminated plate analysis using the natural radial element method

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ABSTRACT

In this work an innovative numerical approach, combining the simplicity of low-order finite elements connectivity with the geometric flexibility of meshless methods, is extended to the elastostatic analysis of composite laminated plates. The Voronoï diagram geometric concept is used to enforce the nodal connectivity and the background integration mesh is constructed uniquely dependent on the computational nodal mesh through the application of the Delaunay triangulation. With the proposed numerical method, the nodal connectivity is imposed through nodal sets with reduced size, reducing significantly the test function construction cost. The interpolations functions are constructed using Euclidian norms and easily obtained. In this work it is considered the first-order plate shear deformation theory. To prove the good behaviour of the proposed interpolation function elastostatic composite laminated plate benchmark examples are solved.

1. Introduction

Within the scientific community, the FEM [1] is a well implemented numerical method, successfully applied to solve a variety of problems in engineering. However the dependence on the computational mesh leads to some limitations, such as the high errors and lost of accuracy verified with the excessive element distortion and/or the solid domain complex geometries. With the objective of eliminating part of the finite element method (FEM) shortcomings, several meshless methods [2,3] were developed in the recent years.

In meshless methods the problem domain can be discretized in a randomly distributed nodal mesh, since meshless methods approximate (or interpolate) the field functions within a flexible influence domain instead of an element as in the FEM. It is the overlap of the interest points influence domains that permits to impose the nodal connectivity and define the field function applicability space.

Several meshless methods were applied with strong form solutions in solid mechanics [4,5], however the present work aims to apply the meshless formulation to the weak form solution since it is more flexible and wide. Being the solution obtained based in the weak formulation meshless methods can be divided in two classes, the approximants meshless methods and the interpolants meshless methods.

Regarding meshless methods with approximation functions, the smooth particle hydrodynamics (SPH) [6], based in the kernel estimation [7], was one of the first to be developed. The diffuse element method (DEM) [8] was the first to use the moving least square approximants in the construction of the approximation function, initially it was proposed for surface fitting [9]. Latter the DEM was improved and consequently the element free Galerkin method (EFGM) was developed [10]. With the introduction of a correction function for the kernel approximation on the SPH, the reproducing kernel particle method [11] was developed, and in the same period the meshless local Petrov–Galerkin method (MLPG) [12] was presented.

One of the major disadvantage of approximation meshless methods is the lack of the delta Kronecker property, difficulting the imposition of essential and natural boundary conditions. Therefore the scientific community started to research and to develop meshless methods using interpolation functions. One of the most popular interpolator meshless method is the natural element method (NEM) [13,14], which uses the Sibson interpolation functions and the Voronoï diagram to impose the nodal connectivity.

Based only on a group of arbitrarily distributed points, the Point Interpolation Method (PIM) [15] constructs a polynomial interpolation with Kronecker delta function property. Later, the addition of a radial basis functions to the basis of the interpolation functions permitted to develop the Radial Point Interpolation Method (RPIM) [16]. More recently, using the advantages of the natural neighbours

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on the imposition of nodal connectivity and the radial point interpolator technique, it was developed the natural neighbour radial point interpolation method (NNRPIM) [17].

In this work a new meshless method, the Natural Radial Element Method (NREM), is extended to the analysis of composite laminated plates. The main advantage of this numerical approach is the combination between the connectivity simplicity of a low-order finite element and the geometric flexibility of a meshless method. The Voronoï diagram [18] concept is used to obtain the NREM nodal connectivity and to establish the Voronoï cells. Then, using the Delaunay triangulation [19], the integration mesh is constructed. With the NREM the integration mesh is completely dependent on the nodal mesh, permitting to classify the NREM as a truly meshless method. This NREM feature permits to define uniquely the computational nodal mesh discretizing the problem domain, and then using well-known mathematical and geometrical concepts the integration mesh is automatically defined, without requiring any additional information or interference from the user. Using the Delaunay triangulation small size influence-domains are determined, each one with only $n = d + 1$ nodes, being d the problem domain dimension, $\Omega \subset \mathbb{R}^d$.

The Euclidean norm basis function (ENBF) is applied to construct the NREM interpolation functions, used as trial functions in the Galerkin weak form. The interpolation function construction process is very similar with the radial point interpolators [16,17], however the ENBF does not require any shape parameter as the radial basis functions used in [16,17].

The NREM extension to the analysis of laminated composite plates is performed considering the first order plate shear deformation theory (FSDT) presented in the early works of Reissner [20] and Mindlin [21], which assumes first order displacement functions and considers a shear correction factor for attenuating the non-zero transverse shear strain on the top and bottom surfaces. In [22] a review on the developments of meshless methods and their applications in the analysis of composite structures is presented. One of the first works to extend meshless methods to the analysis of plates considering the FSDT was the work of Donning and Liu [23]. Since then this solid mechanics problem was analysed by other meshless methods [24–28], in which techniques to avoid the shear-locking phenomenon are extensively described. In the work of Liew et al. [29] laminated composite plates and beams were analysed using the EFGM. Also considering the EFGM, Belinha and Dinis [30,31] presented a non-linear analysis for plates and composite laminates.

This work is divided in five sections. In Section 2 the NREM is presented, the nodal connectivity and the integration mesh determination are described, as well as the NREM interpolation function construction. The Galerkin weak form considering the FSDT for the NREM formulation is presented in Section 3, along with the matricial procedure to obtain the equilibrium equation system. In Section 4 linear elastostatic composite laminated plate benchmark examples are solved and the obtained NREM solution is compared with other numerical methods and the available exact solution. A comparison between the computational effort of the NREM and other numerical method is also presented. The work ends with the conclusions and remarks in Section 5.

2. Natural radial element method

In this section the NREM nodal connectivity and the NREM background integration mesh construction are explained. Next, a detailed description of the construction of the proposed NREM interpolation functions is presented.

The natural neighbours mathematical concept was firstly introduced by Sibson [32] for data fitting and field smoothing. In this work this concept is used to enforce the nodal connectivity. Consider the nodal set $\boldsymbol{N} = \{n_1, n_2, \ldots, n_N\}$ discretizing the space domain $\Omega \subset \mathbb{R}^2$ in $\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N\} \in \Omega$. The Voronoï diagram of $\boldsymbol{N}$ is the partition of the function space discretized by $\boldsymbol{X}$ in sub-regions V_i, closed and convex, Fig. 1a. Each sub-region V_i is associated to the node n_i in a way that any point in the interior of V_i is closer to n_i than any other node $n_j \in \boldsymbol{N} \wedge j \neq i$. The set of Voronoï cells $\boldsymbol{V}$ defines the Voronoï diagram, $\boldsymbol{V} = \{V_1, V_2, \ldots, V_N\}$. The Voronoï cell is defined by, $V_i := \{\boldsymbol{x}_I \in \Omega \subset \mathbb{R}^2 : \|\boldsymbol{x}_I - \boldsymbol{x}_i\| < \|\boldsymbol{x}_I - \boldsymbol{x}_j\|, \forall i \neq j\}$, being $\boldsymbol{x}_I$ an interest point of the domain and $\|\cdot\|$ the Euclidian metric norm. Thus a Voronoï cell V_i is the geometric place where all points are closer to n_i than to any other node. The Voronoï diagrams implications are extensive, with applications from the natural sciences to engineering. A detailed description of the properties and applications of Voronoï diagrams can be found in [33,34] and efficient algorithms to construct Voronoï tessellations are available in [35].

The geometrical dual of the Voronoï diagram is the Delaunay triangulation, which can be obtained by connecting the nodes from Voronoï cells that have common boundaries. The duality between the Voronoï diagram and the Delaunay triangulation implies that a Delaunay edge exists between two nodes in the plane if and only if their Voronoï cells share a common edge. It is the Delaunay property on Voronoï diagrams that permits to construct an integration mesh completely dependent on the nodal mesh discretizing the problem domain.

2.1. Integration mesh

To obtain the elastostatic displacement solution, the NREM uses the weak form of Galerkin to construct the discrete system of equations. Therefore, in order to numerically integrate the equilibrium equations governing the studied physic phenomenon it is required a background integration mesh. Using the Voronoï diagram, of the discretized domain, the integration mesh is constructed, completely dependent on the nodal mesh discretizing the problem domain. With the Voronoï cells determined and the natural neighbours of each node defined is possible to construct an integration mesh [17]. Considering the Voronoï cell V_8, represented in Fig. 1b, formed by the nodal set $\{n_6 n_7 n_9 n_{13} n_{14}\}$ extracted from discretized domain $\Omega \subset \mathbb{R}^2$ in Fig. 1a. It is possible to divide the Voronoï cell V_i in n partitions $A_I^i \subset V_i \subset \Omega$, being n the total number of natural neighbours of n_i and $\bigcup_{I=i}^{n} A_I^i = V_i$. For each partition A_I^i one integration point is determined $\boldsymbol{x}_I$, Fig. 1b. The geometric place of $\boldsymbol{x}_I$ is the barycentre of the partition A_I^i and the integration weight of $\boldsymbol{x}_I$ is obtained with $w_I^i = \int_{A_I^i} d\Omega$. Repeating the proposed procedure for all the Voronoï cells in the Voronoï diagram it is possible to obtain an integration mesh $\boldsymbol{Q} = \{\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_Q\} \in \Omega$ with $\boldsymbol{q}_I \in \mathbb{R}^2$ and $\sum_{I=1}^{Q} w_I = \int_{\Omega} d\Omega$. A detailed description of this procedure can be found in the literature [17].

In addition to the integration scheme presented above, in this work another integration procedure is proposed. Instead of considering the geometric place of each interest point $\boldsymbol{x}_I$ in the barycentre of the partition A_I^i, each interest point is considered coincident with the nearest node. For this case, considering the nodal set presented in Fig. 1b, $\boldsymbol{x}_I = \boldsymbol{x}_{n8}$. To identify this numerical integration procedure the expression "non-centred integration" is used. It was found that with this integration scheme the NREM convergence rate and accuracy were significantly increased and the shear-locking phenomenon was strongly attenuated.

2.2. Nodal connectivity

In the majority of meshless methods, the most common procedure to impose the nodal connectivity is with the overlap of the