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Analysis of composite plates by a unified formulation-cell based smoothed finite element method and field consistent elements

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ABSTRACT

In this article, we combine Carrera's Unified Formulation (CUF) [13,7] and cell based smoothed finite element method [28] for studying the static bending and the free vibration of thin and thick laminated plates. A 4-noded quadrilateral element based on the field consistency requirement is used for this study to suppress the shear locking phenomenon. The combination of cell based smoothed finite element method and field consistent approach with CUF allows a very accurate prediction of field variables. The accuracy and efficiency of the proposed approach are demonstrated through numerical experiments.

1. Introduction

With the rapid development of engineering, there is an increasing demand for new materials suited for harsh working environments. Engineered materials such as composite materials are used in the construction of aeronautical and aerospace vehicles, as well as civil and mechanical structures. This is because of their excellent strength-to and stiffness-to-weight ratios and the possibility of tailoring their properties to optimize the structural response. However, the analysis of such structures is a complex task, compared with conventional single layer metallic structures. This is because of coupling between membrane, torsion and bending strains; weak transverse shear rigidities; and discontinuity of the mechanical characteristics through the thickness of the laminates. For these reasons, accurate modeling and simulating the characteristics of composite structures through different higher-order displacement functions for two-dimensional theories is taking an important part of mechanics and materials research. Indeed two dimensional theories lead to much less expensive models compared to three-dimensional theories. In this context, analytical/numerical methods based on various 2D higher-order theories for static and dynamic analyses of rectangular laminates have been the subject of increasing attention in the research community.

Various structural theories proposed for evaluating the characteristics of composite laminates under different loading situations were reviewed by [38,29,19] and recently by Khanda et al. [21]. In general, three different approaches have been used to study laminated composite structures: single layer theories, discrete layer theories and mixed plate theory. In the single layer theory approach, layers in laminated composites are assumed to be one equivalent single layer (ESL), whereas in the discrete layer theory approach, each layer is considered in the analysis. Although the discrete layer theories provide very accurate prediction of the displacements and the stresses, increasing the number of layers increases the number of unknowns. This can be prohibitively costly and significantly increase the computational time [48]. To overcome the above limitation, zig–zag models developed by Murukami [30] can satisfy the transverse shear stresses continuity conditions at the interfaces. Moreover, the number of unknowns are independent of the number of layers. Reddy and Robbins [42] presented a review of various equivalent-single-layer and layer-wise laminated plate theories and their finite element models.

Recently, some researchers have attempted to combine single layer theories and discrete layer theories to overcome the limitations of each one. Carrera [13,31,7] derived a series of axiomatic approaches, coined as 'Carrera Unified Formulation' (CUF) for the general description of two-dimensional formulations for multilayered plates and shells. With this unified formulation it is possible to implement in a single software a series of hierarchical formulations, thus affording a systematic assessment of different theories,

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ranging from simple ESL models up to higher order layerwise descriptions. This formulation is a valuable tool for gaining a deep insight into the complex mechanics of laminated structures.

The CUF has been used to develop discrete models such as the finite element method (FEM) [13,31], and more recently, meshless methods based upon collocation with radial basis functions [16]. Although the FEM provides a general and systematic technique for constructing basis functions, difficulties still exist in the development of plate elements based on shear deformation theories, one of which is the shear locking phenomenon. Different techniques by which the locking phenomena can be suppressed include: (a) retain the original interpolations and use an optimal integration rule [18]; (b) assumed natural strain method [1,45] and (c) enhanced assumed strain method [44]. Recently, Cinefra et al. employed mixed interpolation of tensorial components (MITC) technique for 4-noded [8] and 9-noded [10–12] multilayered shell/plate elements formulated on the basis of CUF.

Another set of methods have emerged to address the shear locking in the FEM. By incorporating the strain smoothing technique into the finite element method (FEM), Liu et al. [28] have formulated a series of smoothed finite element methods (SFEMs), named as cell-based SFEM (CS-FEM) [34,4], node-based SFEM [26], edge-based SFEM [25], face-based SFEM [33] and α-FEM [24]. And recently, edge based imbricate finite element method (EI-FEM) was proposed in [9] that shares common features with the ES-FEM. As the SFEM can be recast within a Hellinger–Reissner variational principle, suitable choices of the assumed strain/gradient space provides stable solutions. Depending on the number and geometry of the subcells used, a spectrum of methods exhibiting a spectrum of properties is obtained. Interested are referred to the literature [27,34] and references therein. Nguyen-Xuan et al. [37] employed CS-FEM for Mindlin–Reissner plates. The curvature at each point is obtained by a non-local approximation via a smoothing function. From the numerical studies presented, it was concluded that the CS-FEM technique is robust, computationally inexpensive, free of locking and importantly insensitive to mesh distortions. The SFEM was extended to various problems such as shells [35], heat transfer [49], fracture mechanics [36] and structural acoustics [17] among others. In [3], CS-FEM has been combined with the extended FEM to address problems involving discontinuities.

In this study, a $\mathcal{C}^o$ 4-noded quadrilateral element is employed to study the static bending and free vibration of laminated composites. The plate kinematics is based on Carrera Unified Formulation (CUF) and a sinusoidal shear deformation theory is used to approximate the displacements. A CS-FEM with field consistency approach is employed to study the response of laminated composites. The influence of various parameters, viz., the thickness of the plate, the fiber orientation, the ply lay up and the material properties on the response of the laminated composite plates is studied numerically.

The paper is organized as follows. Section 2 presents an overview of the Unified Formulation, the finite element discretization and the cell-based smoothing technique for implementation of the CUF. A discussion on computing the fundamental nuclei is also given. The results of the present formulation are compared with those available in the literature in Section 3, bringing out the influence of various parameters on the static bending and the natural frequencies, followed by concluding remarks in the last section.

2. Carrera unified formulation

2.1. Basis of CUF

Let us consider a laminated plate composed of perfectly bonded layers with coordinates x, y along the in-plane directions and z along the thickness direction of the whole plate, while z_k is the thickness of the kth layer. The CUF is a useful tool to implement a large number of two-dimensional models with the description at the layer level as the starting point. By following the axiomatic modeling approach, the displacements $\mathbf{u}(x,y,z) = (u(x,y,z), v(x,y,z), w(x,y,z))$ are written according to the general expansion as:

$$\mathbf{u}(x,y,z) = \sum_{\tau=0}^{N} F_\tau(z)\mathbf{u}_\tau(x,y) \tag{1}$$

where $F(z)$ are known functions to model the thickness distribution of the unknowns, N is the order of the expansion assumed for the through-thickness behavior. By varying the free parameter N, a *hierarchical* series of two-dimensional models can be obtained. The strains are related to the displacement field via the geometrical relations:

$$\begin{aligned} \boldsymbol{\varepsilon}_{pG} &= \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \gamma_{xy} \end{bmatrix}^{\mathrm{T}} = \mathbf{D}_p\mathbf{u} \\ \boldsymbol{\varepsilon}_{nG} &= \begin{bmatrix} \gamma_{xz} & \gamma_{yz} & \varepsilon_{zz} \end{bmatrix}^{\mathrm{T}} = (\mathbf{D}_{np} + \mathbf{D}_{nz})\mathbf{u} \end{aligned} \tag{2}$$

where the subscript G indicate the geometrical equations, $\mathbf{D}_p$, $\mathbf{D}_{np}$ and $\mathbf{D}_{nz}$ are differential operators given by:

$$\mathbf{D}_p = \begin{bmatrix} \partial_x & 0 & 0 \\ 0 & \partial_y & 0 \\ \partial_y & \partial_x & 0 \end{bmatrix}, \quad \mathbf{D}_{np} = \begin{bmatrix} 0 & 0 & \partial_x \\ 0 & 0 & \partial_y \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{D}_{nz} = \begin{bmatrix} \partial_z & 0 & 0 \\ 0 & \partial_z & 0 \\ 0 & 0 & \partial_z \end{bmatrix}. \tag{3}$$

The 3D constitutive equations are given as:

$$\begin{aligned} \boldsymbol{\sigma}_{pC} &= \mathbf{C}_{pp}\boldsymbol{\varepsilon}_{pG} + \mathbf{C}_{pn}\boldsymbol{\varepsilon}_{nG} \\ \boldsymbol{\sigma}_{nC} &= \mathbf{C}_{np}\boldsymbol{\varepsilon}_{pG} + \mathbf{C}_{nn}\boldsymbol{\varepsilon}_{nG} \end{aligned} \tag{4}$$

with

$$\begin{aligned} \mathbf{C}_{pp} &= \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix} \mathbf{C}_{pn} = \begin{bmatrix} 0 & 0 & C_{13} \\ 0 & 0 & C_{23} \\ 0 & 0 & C_{36} \end{bmatrix} \\ \mathbf{C}_{np} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{13} & C_{23} & C_{36} \end{bmatrix} \mathbf{C}_{nn} = \begin{bmatrix} C_{55} & C_{45} & 0 \\ C_{45} & C_{44} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \end{aligned} \tag{5}$$

where the subscript C indicate the constitutive equations. The *Principle of Virtual Displacements* (PVDs) in case of multilayered plate subjected to mechanical loads is written as:

$$\begin{aligned} &\sum_{k=1}^{N_k} \int_{\Omega_k} \int_{A_k} \left\{ \left(\delta\boldsymbol{\varepsilon}_{pG}^k\right)^{\mathrm{T}} \boldsymbol{\sigma}_{pC}^k + \left(\delta\boldsymbol{\varepsilon}_{nG}^k\right)^{\mathrm{T}} \boldsymbol{\sigma}_{nC}^k \right\} \mathrm{d}\Omega_k \mathrm{d}z \\ &\quad = \sum_{k=1}^{N_k} \int_{\Omega_k} \int_{A_k} \rho^k \delta\mathbf{u}_s^{k^{\mathrm{T}}} \ddot{\mathbf{u}}^k \mathrm{d}\Omega_k \, \mathrm{d}z + \sum_{k=1}^{N_k} \delta\mathbf{L}_e^k \end{aligned} \tag{6}$$

where ρ^k is the mass density of the kth layer, Ω_k, A_k are the integration domain in the (x,y) and the z direction, respectively. Upon substituting the geometric relations (Eq. (2)), the constitutive relations (Eq. (4)) and the unified formulation into the PVD statement, we have:

$$\begin{aligned} &\int_{\Omega_k} \int_{A_k} \left\{ \left(\mathbf{D}_p^k F_s \delta\mathbf{u}_s^k\right)^{\mathrm{T}} \left\{ \mathbf{C}_{pp}^k \mathbf{D}_p^k F_\tau \mathbf{u}_\tau^k + \mathbf{C}_{pn}^k \left(\mathbf{D}_{n\Omega}^k + \mathbf{D}_{nz}^k\right) F_\tau \mathbf{u}_\tau^k \right\} \right. \\ &\left. + \left[\left(\mathbf{D}_{n\Omega}^k + \mathbf{D}_{nz}^k\right) f_x \delta\mathbf{u}_s^k \right)^{\mathrm{T}} \left(\mathbf{C}_{np}^k \mathbf{D}_p^k F_\tau \mathbf{u}_\tau^k + \mathbf{C}_{nn}^k \left(\mathbf{D}_{n\Omega}^k + \mathbf{D}_{nz}^k\right) F_\tau \mathbf{u}_\tau^k \right) \right] \right\} \mathrm{d}\Omega_k \mathrm{d}z \\ &= \sum_{k=1}^{N_k} \int_{\Omega_k} \int_{A_k} \rho^k \delta\mathbf{u}_s^{k^{\mathrm{T}}} \ddot{\mathbf{u}}^k \mathrm{d}\Omega_k \mathrm{d}z + \sum_{k=1}^{N_k} \delta\mathbf{L}_e^k \end{aligned} \tag{7}$$

After integration by parts, the governing equations for the plate are obtained: