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Non-linear static and dynamic analysis of skew sandwich plates

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ABSTRACT

The present work deals with the flexural behavior of thin skew sandwich plates, with different types of facings, subjected to transverse static and dynamic loadings. Based on higher order shear deformation theory and von-Karman's non-linearity, the equations of motion are derived using Hamilton's principle. Transformation from physical to computational domain is carried out using linear transformation and chain rule of differentiation. Fast converging finite double Chebyshev series and Houbolt time marching scheme are used for spatial and temporal discretizations, respectively. Quadratic extrapolation technique is used for linearization of the equations of motion. The effect of skew angle, core thickness, lamination scheme and material properties on the static and dynamic behavior of skew sandwich plate is presented. Transient response of skew sandwich plates subjected to short duration pulse loadings is also obtained.

1. Introduction

The need for high performance and low weight plate/panel structures in aerospace, marine and other engineering structures necessitates the use of sandwich constructions. The analysis of rectangular sandwich plates has drawn a great deal of attention from the research community [1–3]. The behavior of sandwich plates has been analyzed by using different theories e.g. equivalent single layer theories, layer wise theories, refined theories etc. The major problem in the analysis of these structures arises due to variation of in-plane displacement across the thickness and transverse shear strain discontinuity at the boundaries of core and face due to significant difference in transverse shear properties and thickness of the layers. Even higher order shear deformation theories fail to predict the accurate behavior of these types of thick panels. However global higher order theories can be used for the analysis of thin sandwich plates, minimizing the computational cost.

Other than rectangular shapes, skew plates are used extensively in civil, aerospace, naval and other industries. The analysis of skew plates becomes more complicated because of the involvement of the oblique boundaries as well as the coupling among the stiffness coefficients. Different methods have been employed to analyze the behavior of multilayered skew plate's e.g. Finite element method [4], Differential quadrature method [5], Ritz method [6] etc. A series based solution for the static analysis of parallelogram shaped sandwich plates was presented by Kennedy [7]. Monforton and Michail [8] and Ng and Kwok [9] used finite element method to analyze skew sandwich plates. Rao and Valsarajan [10] presented the finite deflection analysis of clamped skew sandwich plates using Galerkin's method. Rao and Valsarajan [11] obtained the results for large deformation of clamped skew sandwich plates using parametric differentiation technique. Using finite element displacement model, Ng and Lam [12] obtained the static and free vibration response of clamped and simply supported skew sandwich plates. Ng and Das [13] obtained the large deflection behavior of clamped skew sandwich plates using Galerkin's method in conjunction with Newton-Raphson technique. Rao and Valsarajan [14] analyzed the large deflection behavior of skew sandwich plates using integral equation approach. Employing the Galerkin's method, Ng and Das [15] obtained the buckling and free vibration response of clamped skew sandwich plates. Qin [16] obtained the non-linear response of skew sandwich plates using boundary element method. Ray et.al. [17] obtained the non-linear static and dynamic behavior of freely supported skew sandwich plates using Banerjee's hypothesis. Qin and Diao [18] obtained the solution for clamped skew sandwich plate resting on elastic foundation using hybrid-Trefftz p-element. Wang et.al. [19] obtained the free vibration response of skew sandwich plates with orthotropic core and laminated facings using p-Ritz method. Makhecha et.al. [20] obtained the finite element based transient response of thick skew sandwich laminated plates subjected to mechanical and thermal loads using higher order shear deformation theory. Chakrabarti and Sheikh [21] presented the bending response of sandwich plates with laminated facings using refined plate theory and finite element method. They also presented the linear static results for clamped and simply supported skew sandwich plates. Garg et.al. [22] presented free vibration response of skew sandwich laminates using higher order shear deformation theory and finite element

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method. Park et.al. [23] obtained the dynamic response of skew sandwich plates using higher order shear deformation theory.

From the literature, it is observed that most of the work is related to the analysis of skew sandwich plates using numerical techniques. Limited attention is paid to study the non-linear displacement response of skew sandwich plates using analytical or semi analytical tools. In the present work, non-linear static and dynamic response of skew sandwich plates with or without laminated facings is presented. Using higher order shear deformation theory and von-Karman's non-linearity, the equations of motion are derived. These equations are then transformed from physical to computational domain using linear transformation and chain rule of differentiation. Fast converging finite double Chebyshev series and Houbolt time marching scheme are used for spatial and temporal discretizations, respectively. The non-linear terms are linearized using quadratic extrapolation technique. Effect of core to face thickness, skin to core material property ratio, skew angle and lamination scheme on the flexural response of skew sandwich plates are studied. The dynamic analysis is carried out for different types of loading conditions including short duration pulses.

2. Mathematical formulation

Based on HSDT with cubic variation of in-plane displacements through the thickness and constant transverse displacement, the displacement field at a point in the plate is expressed as [24];

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) + z\psi_x(x,y,t) + z^2 u_1(x,y,t) + z^3\phi_x(x,y,t)\\
v(x,y,z,t) &= v_0(x,y,t) + z\psi_y(x,y,t) + z^2 v_1(x,y,t) + z^3\phi_y(x,y,t)\\
w(x,y,z,t) &= w_0(x,y,t)
\end{aligned} \tag{1}
$$

Where, u_0, v_0 are the in-plane displacements and w_0 is the transverse displacement of a point (x, y) on the middle plane of the plate, respectively. The functions ψ_x and ψ_y are rotations of the normal to the middle plane about y and x axes, respectively. The parameters u_1, v_1, ϕ_x and ϕ_y are the higher order terms in the Taylor's series expansion, representing higher-order transverse cross-sectional deformation modes.

The stress-strain relations for kth layer in the sandwich plate are written as;

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}_k \tag{2}
$$

where $\overline{Q}_{ij}$ for i,j = 1, 2, 4, 5, 6 are transformed reduced stiffness coefficients. Employing von-Karman non-linear kinematics and using the displacement field in Eq. (1), strain-displacement relations are expressed as;

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \gamma_{yz}^o \\ \gamma_{xz}^o \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ 2v_1 \\ 2u_1 \end{Bmatrix} + z^2\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \\ 3\phi_y \\ 3\phi_x \end{Bmatrix} + z^3\begin{Bmatrix} \kappa_x^1 \\ \kappa_y^1 \\ \kappa_{xy}^1 \\ 0 \\ 0 \end{Bmatrix} \tag{3.1}
$$

where

$$
\begin{aligned}
&\varepsilon_x^o = u_{o,x} + 0.5(w_{o,x})^2; \quad \varepsilon_y^o = v_{o,y} + 0.5(w_{o,y})^2;\\
&\gamma_{xy}^o = u_{o,y} + v_{o,x} + w_{o,x}w_{o,y}\\
&\gamma_{yz}^o = \psi_y + w_{o,y}; \quad \gamma_{xz}^o = \psi_x + w_{o,x}; \quad \kappa_x = \psi_{x,x}; \quad \kappa_y = \psi_{y,y};\\
&\kappa_{xy} = \psi_{x,y} + \psi_{y,x}\\
&\varepsilon_x^1 = u_{1,x}; \quad \varepsilon_y^1 = v_{1,y}; \quad \gamma_{xy}^1 = u_{1,y} + v_{1,x}; \quad \kappa_x^1 = \phi_{x,x}; \quad \kappa_y^1 = \phi_{y,y};\\
&\kappa_{xy}^1 = \phi_{x,y} + \phi_{y,x}
\end{aligned} \tag{3.2}
$$

The in-plane stress and moment resultants are expressed as:

$$
\begin{Bmatrix} [N] \\ [M] \\ [N^*] \\ [M^*] \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [D] & [E] \\ [B] & [D] & [E] & [F] \\ [D] & [E] & [F] & [H] \\ [E] & [F] & [H] & [J] \end{bmatrix} \begin{Bmatrix} [\varepsilon^0] \\ [\kappa] \\ [\varepsilon^1] \\ [\kappa^1] \end{Bmatrix} \tag{4.1}
$$

Transverse shear stress resultants are written as:

$$
\begin{Bmatrix} Q_y \\ Q_x \\ S_y \\ S_x \\ Q_y^* \\ Q_x^* \end{Bmatrix} = \begin{bmatrix} [\overline{A}] & [\overline{B}] & [\overline{D}] \\ [\overline{B}] & [\overline{D}] & [\overline{E}] \\ [\overline{D}] & [\overline{E}] & [\overline{F}] \end{bmatrix} \begin{Bmatrix} \psi_y + w_{0,y} \\ \psi_x + w_{0,x} \\ 2v_1 \\ 2u_1 \\ 3\phi_y \\ 3\phi_x \end{Bmatrix} \tag{4.2}
$$

Where,

$$[N] = [\,N_x \quad N_y \quad N_{xy}\,]^T; \quad [M] = [\,M_x \quad M_y \quad M_{xy}\,]^T$$

$$[N^*] = [\,N_x^* \quad N_y^* \quad N_{xy}^*\,]^T; \quad [M^*] = [\,M_x^* \quad M_y^* \quad M_{xy}^*\,]^T$$

$$[\varepsilon^0] = [\,\varepsilon_x^0 \quad \varepsilon_y^0 \quad \gamma_{xy}^0\,]^T; \quad [\kappa] = [\,\kappa_x \quad \kappa_y \quad \kappa_{xy}\,]^T$$

$$[\varepsilon^1] = [\,\varepsilon_x^1 \quad \varepsilon_y^1 \quad \gamma_{xy}^1\,]^T; \quad [\kappa^1] = [\,\kappa_x^1 \quad \kappa_y^1 \quad \kappa_{xy}^1\,]^T$$

$[A]$, $[B]$, $[D]$, $[E]$, $[F]$, $[H]$, $[J]$, $[\overline{A}]$, $[\overline{B}]$, $[\overline{D}]$, $[\overline{E}]$, $[\overline{F}]$, the plate stiffness coefficients matrices are defined as:

$$(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}, J_{ij}) = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k} \overline{Q}_{ij}^{(k)}(1, z, z^2, z^3, z^4, z^5, z^6)dz;$$

$$(i,j = 1,2,6)$$

$$(\overline{A}_{ij}, \overline{B}_{ij}, \overline{D}_{ij}, \overline{E}_{ij}, \overline{F}_{ij}) = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k} \overline{Q}_{ij}^{(k)}(1, z, z^2, z^3, z^4)dz; \quad (i,j = 4,5)$$

The governing equations of motion are obtained using the Hamilton's principle and expressed as:

$$
\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1\frac{\partial^2 u_0}{\partial t^2} + I_2\frac{\partial^2 \psi_x}{\partial t^2} + I_3\frac{\partial^2 u_1}{\partial t^2} + I_4\frac{\partial^2 \phi_x}{\partial t^2}\\
&\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_1\frac{\partial^2 v_0}{\partial t^2} + I_2\frac{\partial^2 \psi_y}{\partial t^2} + I_3\frac{\partial^2 v_1}{\partial t^2} + I_4\frac{\partial^2 \phi_y}{\partial t^2}\\
&\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} + q = I_1\frac{\partial^2 w_0}{\partial t^2}\\
&\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2\frac{\partial^2 u_0}{\partial t^2} + I_3\frac{\partial^2 \psi_x}{\partial t^2} + I_4\frac{\partial^2 u_1}{\partial t^2} + I_5\frac{\partial^2 \phi_x}{\partial t^2}\\
&\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I_2\frac{\partial^2 v_0}{\partial t^2} + I_3\frac{\partial^2 \psi_y}{\partial t^2} + I_4\frac{\partial^2 v_1}{\partial t^2} + I_5\frac{\partial^2 \phi_y}{\partial t^2}\\
&\frac{\partial N_x^*}{\partial x} + \frac{\partial N_{xy}^*}{\partial y} - 2S_x = I_3\frac{\partial^2 u_0}{\partial t^2} + I_4\frac{\partial^2 \psi_x}{\partial t^2} + I_5\frac{\partial^2 u_1}{\partial t^2} + I_6\frac{\partial^2 \phi_x}{\partial t^2}\\
&\frac{\partial N_y^*}{\partial y} + \frac{\partial N_{xy}^*}{\partial x} - 2S_y = I_3\frac{\partial^2 v_0}{\partial t^2} + I_4\frac{\partial^2 \psi_y}{\partial t^2} + I_5\frac{\partial^2 v_1}{\partial t^2} + I_6\frac{\partial^2 \phi_y}{\partial t^2}\\
&\frac{\partial M_x^*}{\partial x} + \frac{\partial M_{xy}^*}{\partial y} - 3Q_x^* = I_4\frac{\partial^2 u_0}{\partial t^2} + I_5\frac{\partial^2 \psi_x}{\partial t^2} + I_6\frac{\partial^2 u_1}{\partial t^2} + I_7\frac{\partial^2 \phi_x}{\partial t^2}\\
&\frac{\partial M_y^*}{\partial y} + \frac{\partial M_{xy}^*}{\partial x} - 3Q_y^* = I_4\frac{\partial^2 v_0}{\partial t^2} + I_5\frac{\partial^2 \psi_y}{\partial t^2} + I_6\frac{\partial^2 v_1}{\partial t^2} + I_7\frac{\partial^2 \phi_y}{\partial t^2}
\end{aligned} \tag{5.1-5.9}
$$

where $(I_1, I_2, I_3, I_4, I_5, I_6, I_7) = \int_{-h/2}^{h/2}\rho(z)(1, z, z^2, z^3, z^4, z^5, z^6)dz$ are the inertia coefficients.