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Review

Forced vibration analysis of functionally graded beams using nonlocal elasticity

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A B S T R A C T

A forced vibration analysis of functionally graded (FG) nanobeams is considered based on the nonlocal elasticity theory. The solution is obtained by using Navier method for various shear deformation theories. The material properties of the FG nanobeam vary through the thickness direction according to a simple power law. Effects of the nonlocal parameter, different material composition and length-to-thickness ratio of considered element on the vibration and the effect of frequency ratio and different dynamic loading conditions on dimensionless maximum deflection and mode shapes of FG nanobeam are investigated. As a result the dynamic behavior of the FG nanobeam is influenced by the nonlocal effects. The dynamic deflections obtained by the classical (local) theory are smaller than obtained by the nonlocal theory due to the nonlocal effects.

Contents

1. Introduction

The free vibration analysis of structural elements is a common study as important as among all engineering problems and knowledge of the natural frequencies suggests the designer avoid the peak resonances which occur nearby the natural frequencies [1]. Also, dynamic systems are often subjected to time-dependent external forces leading to the forced vibration whose amplitude depends on the frequency ratio. If the frequency of the external force coincides with one of the natural frequencies of considered element such as strings, rods, membranes, beams, plates and shells, resonance occurs, which leads to dangerously large oscillations. Hence, in the case of forced vibration it is important to know the behavior of considered element nearby the resonance condition. This view constitutes the main object of this study.

The carbon nanotubes (CNTs) are invented by Ijima [2]. The studies related with CNTs show that these structures have good mechanical properties [3–7]. Hence nanostructures attract great attention by researchers based on molecular dynamics and continuum mechanics. However, due to large number of equations in the molecular dynamics, the nonlocal theory of Eringen [8–11] which is one of the size-dependent continuum mechanics models is widely used, recently. Also, this theory provides to solution of problems which include the large nano-scale structures. The theory assumes that the stress at a point is a function of strains not only at that point as in the classical elasticity but also at all points in the continuum. Meanwhile, nonlocal theories consider the forces between atoms and the internal length scale which is in the constitutive equations as a material parameter [12]. Due to the difficulties in the molecular dynamics, the nonlocal theory of Eringen is preferred by some of

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researchers to investigate the elastic behaviors of single walled, double walled and multi-walled CNTs like static [13–18], buckling [19–28], free vibration [29–36] wave propagation [37–40] and force vibration analysis [41,42]. Also, in some of the studies related with CNTs, these structures are assumed as a cylindrical shell model and as a beam model [43–47]. Vibration of CNTs embedded in an elastic medium was considered by some researchers [48–51] and axial vibration of CNTs was investigated by some researchers [52–55] with the nonlocal theory of Eringen. Filiz and Aydogdu [56] investigated the small scale effects on axial vibrations of heterojunction CNTs based on the local and nonlocal rod theories. Simsek [57] solved the free longitudinal vibration of axially FG tapered nanorods with nonlocal effects.

Reddy and Pang [58] reformulated the Euler–Bernoulli and Timoshenko beam theories using the nonlocal differential constitutive relations of Eringen. Numerical results were presented to bring out the effect of nonlocal behaviors on deflections, buckling loads and natural frequencies of CNTs. Nonlocal results for bending, buckling and vibration of nanobeams were obtained by Reddy [59] applying Euler–Bernoulli, Timoshenko, Reddy and Levinson beam theories, by Aydogdu [60] applying a generalized nonlocal beam theory, by Thai [61] applying a refined theory, by Thai and Vo [62] applying a nonlocal sinusoidal shear deformation theory of Touratier [63]. Eltaher et al. [64] investigated free vibration of nanobeams using finite element method. Free vibration analysis and static and stability analysis of FG size-dependent nanobeams were presented by Eltaher et al. [65] using finite element method. Free and forced axial vibrations of damped nonlocal rods were investigated by Adhikari et al. [66]. A frequency-dependent dynamic finite element method was developed to obtain the forced vibration response. Simsek and Yurtcu [67] presented static bending and buckling of FG nanobeams using nonlocal Timoshenko beam theory. Also, Simsek and Kocatürk [68] considered the forced vibration of FG nanobeam which occurs due to a moving harmonic load using classical elasticity and Simsek [69] considered the forced vibration of single-walled CNTs using nonlocal elasticity theory. According to these results, dynamic deflection of these structures increases with increasing nonlocal parameter.

Aksencer and Aydogdu [70,71] investigated the nonlocal effects on free vibration and buckling and forced vibration of nanoplates with analytical solution. The results show that the dynamic behavior of the nanoplates is greatly influenced by the nonlocal effects. The dynamic deflections predicted by the classical theory are always smaller than those predicted by the nonlocal theory due to the nonlocal effects.

In this study, analytical solutions of free and forced vibration of FG nanobeams are presented using generalized beam theory. The effects of the material composition (p index), the length-to-thickness ratio (L/h) and frequency ratio (Δr) on the vibration frequency of considered nanobeam with different loading conditions are investigated. The mode shapes giving information for geometrical character of the vibration behavior are plotted for considered nanobeams.

2. Nonlocal formulation of FG nanobeams

Nonlocal elasticity theory which overcomes in cases that insufficient of classical elasticity is developed by Eringen. According to nonlocal elasticity theory, differently from the classical elasticity theory the stress field at a point **x** in an elastic continuum not only depends on the strain field at the same point but also on strains at all other points of the body. The linear constitutive equation of the nonlocal, anisotropic elastic solid is expressed as follows

$$\sigma_{kl} = \int_{\mathcal{V}} \lambda_{ijkl}(x', x) e_{ij}(x') d\upsilon' \tag{1}$$

For the homogeneous media, λ_{ijkl} is symmetric function of $x' - x$, e.g.

$$\lambda_{ijkl} = \lambda_{ijkl}(x' - x) = \lambda_{ijkl}(x - x') = \lambda_{jikl}(x' - x) = \lambda_{ijlk}(x' - x) \tag{2}$$

And for homogeneous and isotropic solids it is isotropic function of $\kappa = x' - x$, i.e.

$$\lambda_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \lambda_1(\kappa_i\kappa_j\delta_{kl} + \kappa_k\kappa_l\delta_{ij}) + \lambda_1\kappa_i\kappa_j\kappa_k\kappa_l \tag{3}$$

where the material moduli λ, μ, λ_1, and λ_2 are functions of $\kappa = |\kappa| = |x' - x|$, e.g.

$$\lambda = \lambda(|x' - x|) \tag{4}$$

Hence, the constitutive equation of the nonlocal, isotropic elastic solid is expressed as follows:

$$\sigma_{kl} = \int_{\mathcal{V}} [\lambda(|x' - x|) e_{rr}(x')\delta_{kl} + 2\mu(|x' - x|) e_{rr}(x') e_{kl}(x')] d\upsilon' \tag{5}$$

These constitutive equations corresponds to the constitutive equations of the classical (local) elasticity, by letting

$$\{\lambda(\kappa), \mu(\kappa)\} \to \{\lambda_0, \mu_0\}\delta(\kappa) \tag{6}$$

where $\delta(\kappa)$ is the Dirac-delta measure.

Due to the interatomic attractions die out with distance, the material functions λ and μ must attenuate rapidly with distance, i.e.

$$\lim_{\kappa\to\infty}\{\lambda(\kappa), \mu(\kappa)\} \to 0 \tag{7}$$

For simplification we assume that the degree of attenuation for all material moduli is the same, i.e.

$$\frac{\lambda(|x' - x|)}{\lambda_0} = \frac{\mu(|x' - x|)}{\mu_0} = \alpha(|x' - x|) \tag{8}$$

where λ_0 and μ_0 are the material constants of the local (classical) theory, i.e.

$$t_{kl} = \lambda_0 e_{rr}\delta_{kl} + 2\mu_0 e_{kl} \tag{9}$$

Here, t_{kl} is the Hookean stress tensor and e_{kl} is the strain tensor

$$e_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}) \tag{10}$$

The macroscopic stress **t** at a point **x** in a Hookean solid is related to the strain ε at the point by the generalized Hooke's law

$$\sigma(x) = C(x) : \varepsilon(x) \tag{11}$$

where **C** is the fourth-order elasticity tensor and: denotes the "double-dot product". The Kernel function $\alpha(|\mathbf{x}' - \mathbf{x}|)$ is normalized over the volume of the body, i.e.

$$\int_{\mathcal{V}} \alpha(|x'|) d\upsilon' = 1 \tag{12}$$

With these, constitutive Eq. (5) is abbreviated to

$$\sigma_{kl} = \int_{\mathcal{V}} \alpha(|x' - x|) t_{kl}(x') d\upsilon' \tag{13}$$

A more useful case involves matching the Fourier transforms of constitutive moduli (such as λ_{ijkl}) in the wave number space with the dispersion curves based on the atomic models. The Fourier transform of $\alpha(|\mathbf{x}' - \mathbf{x}|)$ is expressed as follow

$$\bar{\alpha} = (1 + (l\tau)^2 k^2 + \gamma^4 k^4)^{-1} \tag{14}$$

Then the nonlocal stress constitutive Eq. (13), for the infinite media, gives

$$(1 + (l\tau)^2 k^2 + \gamma^4 k^4)\bar{\sigma}_{kl} = \bar{t}_{kl} \tag{15}$$

where $\bar{\sigma}_{kl}$ and $\bar{t}_{kl}$ are the Fourier transforms of σ_{kl} and t_{kl}.