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Static and dynamic analysis of thick laminated plates using enriched macroelements

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ABSTRACT

The development, computational implementation and application of polynomially-enriched plate macro-element are presented in this work. This macro-element has been formulated by the authors for thin isotropic plates using Gram–Schmidt orthogonal polynomials as enrichment functions and, in this work, the first-order shear deformation theory and the material anisotropy is incorporated. For taking into account plates of several geometrical shapes, an arbitrary quadrilateral laminate is mapped onto a square basic one, so that a unique macro-element can be constructed. The obtained formulation is applied to the static and dynamic analysis of thick composite laminated plates. Besides, it is possible to study generally coplanar plate assemblies by combining two or more macro-elements via a special connectivity matrix. Thus, hierarchically enriched global stiffness matrix, mass matrix, and loading vector of general laminated plate structure are derived. Several different boundary conditions may be arranged in the analysis. This procedure gives a matrix equation of static equilibrium and a matrix-eigenvalue problem that can be solved with optimum efficiency. Numerical obtained results show very good correlation with published results. Besides, the formulation produces stable results and it is computationally efficient.

1. Introduction

Composite structures, specially laminated composite plates, are increasingly used in many engineering fields such as civil, marine and aerospace structures, because of their high strength and stiffness to weight ratios. Laminated composite plates allow the controllability of the structural properties through changing the fibre orientation angles, the number of plies and selecting proper composite materials. With the wide use of composite structures in modern industries, mechanical analysis of plates of complex geometry becomes a relevant topic. The solutions to the plate problems are strongly dependent on the geometrical shapes, boundary conditions and material properties. It is widely recognized that closed form solutions are possible only for a few specific cases. The determination of classical solutions (exact and/or approximate) which correspond to the static and dynamic behaviour of anisotropic plates of different shapes and configurations has been studied and is well documented. The bending of anisotropic plates subjected to different normal loads and boundary conditions has been extensively studied (see, for instance, Refs. [1–3]). There are several complete reviews on static behaviour of composite and sandwich plates. Moleiro et al. [4] and Auricchio et al. [5] presented static analysis of square plates using a mixed first-order shear deformation theory into a finite element model. Wang et al. [6] analyzed the static and dynamic behaviour of rectangular plates via FSDT meshless method. Other authors also developed different alternative solutions for rectangular anisotropic plates employing the first order and a refined zigzag theory [7], or various shear deformation theories together with meshless methods [8]. Ferreira et al. [9] presented static and vibration analysis of laminated composite plates using FSDT based on a high order collocation method. The available literature shows that, comparatively, most studies for static and dynamic analysis of laminated plates are mainly concerned with rectangular ones. However, plates of general shape are common industrial elements in many engineering fields like air craft wings, ship substructures, bridge entrance and vehicle bodies. Nevertheless, studies on general quadrilateral plates or polygonal plates with unequal side lengths are rather limited. For these reasons, this work presents a polynomially-enriched plate macro-element and its application to the static and free vibration analysis of moderately thick composite laminated plates of several geometries. The enriched finite macro-element is obtained using Gram–Schmidt orthogonal polynomials, while the different geometrical shapes are represented by the mapping of a square laminated plate defined in terms of its natural coordinates. The formulation presented in this work allows investigating the static and dynamic behaviour of several composite laminated plates with any combination of boundary conditions. Furthermore,

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the described assembly process enables to encompass plates of more complex geometry. Finally, to demonstrate the validity and efficiency of the proposed method, numerical examples are solved and some of them are verified with results from others authors.

2. Mathematical formulation

Let us consider a thick laminated composite plate with an arbitrary-shaped quadrilateral planform as shown in Fig. 1. The laminate is of uniform thickness h and is made up of a number of layers each consisting of unidirectional fibre reinforced composite material. The fibre angle of the kth layer counted from the surface $z = -h/2$ is β, measured from the x axis to the fibre orientation, with all laminate having equal thicknesses. Symmetric lamination of plies is considered in this work.

Based on the First-order Shear Deformation Theory (FSDT) [10,11] the displacement field of a laminated composite plate is expressed as:

$$\begin{aligned} u(x,y,z) &= z\phi_x(x,y) \\ v(x,y,z) &= z\phi_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where (u, v, w) are the displacements of a generic point (x, y, z) in the laminate, w_0 is the displacement of a corresponding point on the mid-plane, and (ϕ_x, ϕ_y) denote the rotations of the transverse normal about y and x axis, respectively.

2.1. Transformation of coordinates

Some authors have used the mapping technique, as commonly employed in finite element analysis, in conjunction with other methods to study the mechanical behaviour of plates of various geometrical shapes. Nallim et al. [12] combined the mapping technique and the Ritz method to derive a general formulation for the analysis of symmetrically laminated plates. Also, Nallim and Oller [13] extended that previous work together with the mapping technique to the general case of unsymmetrically laminated plates. Applying the same concept, an arbitrarily shaped quadrilateral plate in Cartesian coordinates, may be expressed simply by mapping a parent square plate, which will be called master plate, defined in the natural coordinates by the simple boundary equations $\xi = \pm 1$ and $\eta = \pm 1$ (Fig. 2).

The mapping of the Cartesian coordinate system is given by [14,15]:

$$\begin{aligned} x &= \sum_{i=1}^{4} N_i(\xi,\eta)x_i \\ y &= \sum_{i=1}^{4} N_i(\xi,\eta)y_i \end{aligned} \tag{2}$$

where (x_i, y_i), $i = 1, \ldots, 4$ are the coordinates of the four corners of the quadrilateral region R and $N_i(\xi, \eta)$ are the interpolation functions of the serendipity family given by:

$$N_i(\xi,\eta) = \frac{1}{4}(1+\eta_i\eta)(1+\xi_i\xi) \tag{3}$$

The transformation (2) maps a point (ξ, η) in the master plate $\overline{R}$ onto a point (x, y) in the real plate domain R and vice versa if the Jacobian determinant of the transformation given by:

$$|\mathbf{J}| = \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi} \tag{4}$$

is positive.

Applying the chain rule of differentiation it can be shown that the first derivatives of a function in both spaces are related by:

$$\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} = \mathbf{J}^{-1} \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{J_{22}}{|\mathbf{J}|} & -\frac{J_{12}}{|\mathbf{J}|} \\ -\frac{J_{21}}{|\mathbf{J}|} & \frac{J_{11}}{|\mathbf{J}|} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} \tag{5}$$

where $\mathbf{J}$ is the Jacobian given by:

$$\mathbf{J} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \sum x_i N_{i,\xi} & \sum y_i N_{i,\xi} \\ \sum x_i N_{i,\eta} & \sum y_i N_{i,\eta} \end{bmatrix} \tag{6}$$

The elemental area $dxdy$ in the Cartesian domain R is transformed into $|\mathbf{J}|d\xi d\eta$.

2.2. Approximating functions

The use of Gram–Schmidt orthogonal polynomials to study anisotropic plates is very satisfactory, as has been demonstrated by Nallim et al. [12,13,16], since the convergence of the solution is rapid and practically without oscillations. For this reason, in the present paper the transverse deflection and the rotations are expressed in terms of the natural coordinates system by sets of polynomials $\{p_i(\xi)\}$ and $\{q_j(\eta)\}$, of which the first two polynomials are Hermite polynomials and then an adequate number of Gram–Schmidt polynomials are added to formulate a polynomially-enriched plate macro-element. This macro-element has been formulated by the authors for thin plates [17].

Then, the components of displacement field can be expressed as:

$$\begin{aligned} w(\xi,\eta) &= \sum_{i,j=1}^{n} c_{ij}^{w} p_i(\xi) q_j(\eta) \\ \phi_x(\xi,\eta) &= \sum_{i,j=1}^{n} c_{ij}^{\phi x} p_i(\xi) q_j(\eta) \\ \phi_y(\xi,\eta) &= \sum_{i,j=1}^{n} c_{ij}^{\phi y} p_i(\xi) q_j(\eta) \end{aligned} \tag{7}$$

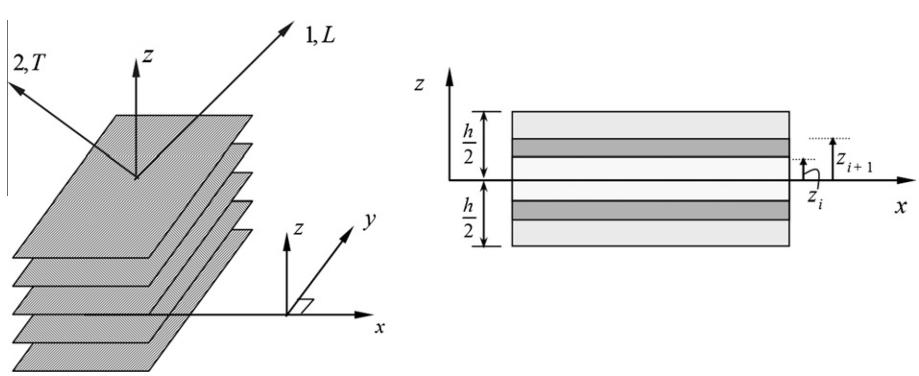

Fig. 1. Geometry of an N-layered symmetric laminate.