Composite Structures 101 (2013) 243–254

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Analytical post-buckling model of corrugated board panels using digital image correlation measurements

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ARTICLE INFO

Article history:
Available online 8 February 2013

Keywords:
Sandwich structures
Post-buckling behaviour
Digital image correlation
Packaging performance optimisation

ABSTRACT

The optimisation of board packages often rely on their load bearing capacity. Then it seems attractive to measure how such thin-walled structures deform using for instance kinematic field measurement techniques, and to incorporate, at least partially, the gained kinematic information within mechanical models. Digital Image Correlation (DIC) can provide a vivid description of the buckling of box panels, e.g. during box compression tests. Therefore, we propose an analytical plate model to predict the elastic post-buckling behaviour of corrugated board box panels where the kinematic boundary conditions emanate from DIC measurements. Comparing experimental and calculated strain fields on the outer liner of board panels as well as box compression force lend some confidence to the model. Further results reveal the heterogeneity of in-plane forces, bending and twisting moments the box panels have to withstand as well as strain fields in usually inaccessible regions of the panels such as the inner liner. Thereby an improvement of the structure of box panels can be envisaged.

1. Introduction

Thanks to its renewable nature, its recyclability and its versatility, the corrugated board is today the most used stiff packaging material. From a mechanical point of view, the corrugated board can be considered as a thin-walled sandwich structure which forms parts of larger packaging structures, i.e. boxes. The latter can also be described as a combination of flat panels joined by regions intentionally damaged, i.e. the box ridges [1]. During their lifetime, boxes are subjected to a large range of mechanical loadings. The most significant is definitely the compression loading. As a consequence, the compression behaviour of corrugated board boxes was largely investigated in standard conditions; i.e. an empty box is compressed between two parallel plates at a constant velocity [2–4]. Buckling phenomena were observed at the scale of panels [5], at the scale of liner sheets [6] or scarcely at the scale of boxes if the latter have a high slenderness. Damage was identified in the form of wrinkles that emerged near corners leading to box collapse [5]. As the box compression involves a significant number of uncontrolled variables that result from the creasing and folding operations, a special compression setup was developed in [7] to test single panels and impose simply supported conditions at edges. The displacement field related to the panel compression was measured by using optical full-field measurement methods [8,9]. By performing measurements of the out-of-plane displacement onto the surface of the both liners, a change in the panel thickness was emphasised [10]. The Digital Image Correlation (DIC) technique was also used to measure the kinematic fields directly on the outside surface of a compressed box [11]. The field analysis revealed strain and stress states highly heterogeneous onto the surface of the outer liner.

At the same time, on the basis of these observations, modelling approaches were developed to predict the box compression behaviour. The complexity of the problem led researchers to make some simplifications regarding the descriptions of the structures of boxes and corrugated layers, as well as the mechanical behaviour of box components. Boxes were first seen as a set of individual edgewise compression loaded and simply supported elastic plates [2,12]. The panel post-buckling behaviour was assumed elastic and modelled analytically [13] or numerically using finite element methods [14,12]. The plate mechanical stiffness constants were obtained by using homogenisation methods [15–18]. Discretisations of the complete board structure by finite elements were also developed [8,4]. Furthermore, a model was proposed to predict local buckling of the liner between the fluting crests [6]. Note also that, in spite of their importance, the behaviour of box ridges was only anecdotally modelled [3].

Recent displacement or strain full-field measurements on the box surface provide accurate data which may help validating and improving the current modelling approaches. For instance, measured fields actually allow boundary conditions in the vicinity or

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0263-8223/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2013.01.023

along panel edges as well on panel surfaces to be refined, and to be more realistic. Thus, we propose in this study to predict the elastic post buckling behaviour of box panels using an analytical approach coupled with boundary conditions that emanate from experimentally measured kinematic fields during a box compression test. To account for these conditions, as the existing analytical approach [13] does not allow specific boundary conditions to be set along edges, a new approach has been developed based on studies aiming at predicting the post-buckling behaviour of plates using a semi-energy finite strip approach [19,20].

In the first section, some preliminary results are described. In the second section, the effective elastic stiffnesses are presented, the displacement fields are formulated, the boundary displacement parameters are assessed and the predicted displacement fields are exposed. In a third section, the predicted kinematic and stress fields on the surface of the outer liner and on the inner liner as well as the membrane forces and bending moments on box panel sections are analysed in order to provide some evidences for optimising boxes which are discussed in a fourth section.

2. Preliminary results

In a previous study [11], the kinematic fields on the panel surfaces of a G-flute corrugated board box with dimensions $73 \times 73 \times 73\ \text{mm}^3$ related to the box compression were measured using an image stereocorrelation method. The G-flute corrugated board is a structure constituted of three layers: two liners and an intercalated corrugated layer called the fluting. It was observed that the box panels are not flat before the compression stage: a slight out-of-plane displacement resulting from the box manufacturing operations was measured. Consequently, three possible panel configurations have to be considered: an undeformed configuration, noted (o), which is only virtual, an initial deformed or prestressed configuration, noted (i), which corresponds to the actual configuration of the box after the manufacturing operation and a final configuration, noted (f), which corresponds to the compressed configuration of the box panels at the maximum recorded axial load. These configurations as well as the typical compression curve are depicted in Fig. 1. Note that the initial deformed configuration (i) can be also considered as a prestressed configuration [21].

The coordinates of a material point in the undeformed configuration are described according to $\underline{X}(X,Y,Z)$ in the reference coordinate system $(\underline{\mathbf{e}}_X, \underline{\mathbf{e}}_Y, \underline{\mathbf{e}}_Z)$. The spatial positions of the material point in the prestressed configuration (i) and in the compressed configuration (f) may be obtained as functions of its material coordinates in the reference coordinate system: $\underline{\mathbf{x}} = \underline{\mathbf{x}}(X,Y,Z)$ and $\underline{\mathbf{y}} = \underline{\mathbf{y}}(X,Y,Z)$, respectively. $\underline{\mathbf{y}}$ can be written as follows:

$$\underline{\mathbf{y}} = \underline{\mathbf{x}} + \underline{\mathbf{u}}^{(c)} = \underline{\mathbf{X}} + \underline{\mathbf{u}}^{(i)} + \underline{\mathbf{u}}^{(c)} = \underline{\mathbf{X}} + \underline{\mathbf{u}}^{(f)}, \tag{1}$$

where $\underline{\mathbf{u}}^{(i)} = \underline{\mathbf{u}}^{(i)}(X,Y,Z)$ is the displacement field between the undeformed configuration (o) and the prestressed configuration (i) expressed as a function of the reference material coordinates, $\underline{\mathbf{u}}^{(c)} = \underline{\mathbf{u}}^{(c)}(X,Y,Z)$ is the displacement field related to the compression (i.e. the transformation between the prestressed configuration (i) and the compressed configuration (f)) also expressed as a function of the reference material coordinates and $\underline{\mathbf{u}}^{(f)}$ the total displacement field.

Two kinds of panels compose a box structure: the "A" panel attached to the "outside" flaps and the "B" panel attached to the "inside" flaps. They are oriented in the planes $(\underline{\mathbf{e}}_X, \underline{\mathbf{e}}_Y)$ and $(\underline{\mathbf{e}}_Y, \underline{\mathbf{e}}_Z)$, respectively, as shown in Fig. 1. The displacement fields $\underline{\mathbf{u}}^{(i)}$, $\underline{\mathbf{u}}^{(c)}$ or $\underline{\mathbf{u}}^{(f)}$ in the "A" panels can be expressed as

$$\underline{\mathbf{u}}^{(x)} = u^{(x)}\underline{\mathbf{e}}_X + v^{(x)}\underline{\mathbf{e}}_Y + w^{(x)}\underline{\mathbf{e}}_Z, \tag{2}$$

where $x = i$, c or f.

Note that the displacement field was measured in windows smaller than the panel surface. Indeed strips of 1.5 mm wide close to the vertical and horizontal ridges were not analysed (see the analysed red and yellow windows depicted in Fig. 1).

Fig. 1 displays the components of the displacement fields of the outer liner of the "A" panel noted $\underline{\mathbf{u}}_{ou}^{(i)}$ and $\underline{\mathbf{u}}_{ou}^{(c)}$. It has to be underlined that the in-plane components of $\underline{\mathbf{u}}_{ou}^{(i)}$ are assumed equal to zero, whereas the out-of-plane component reveals a significant non-flatness. The maximum displacement is indeed recorded at the panel centre and the panel describes a half-wave in the $\underline{\mathbf{e}}_X$-direction as well as in the $\underline{\mathbf{e}}_Y$-direction. The components of the field $\underline{\mathbf{u}}_{ou}^{(c)}$ confirms that buckling is the predominant phenomenon and reveals that the vertical ridges do not remain straight as they deform in-plane and out-of-plane. This last result clearly shows that the simply supported conditions on the panel boundaries are not verified, contrary to what was previously stated in several studies, e.g. [12].

In order to calculate the strain field related to compression, we first express the complete Green–Lagrange strain field (i.e. related to the displacement field between the undeformed configuration and the final configuration). Considering the expression of the deformation gradient tensor $\underline{\underline{\mathbf{F}}}^{(f)}$:

$$\underline{\underline{\mathbf{F}}}^{(f)} = \frac{\partial \underline{\mathbf{y}}}{\partial \underline{\mathbf{X}}} = \underline{\underline{\mathbf{Grad}_X \mathbf{y}}} = \underline{\underline{\mathbf{I}}} + \underline{\underline{\mathbf{H}}}^{(i)} + \underline{\underline{\mathbf{H}}}^{(c)} = \underline{\underline{\mathbf{I}}} + \underline{\underline{\mathbf{H}}}^{(f)}, \tag{3}$$

where $\underline{\underline{\mathbf{I}}}$ is the unit tensor, $\underline{\underline{\mathbf{H}}}^{(i)}$ and $\underline{\underline{\mathbf{H}}}^{(c)}$ are the gradients of the displacement $\underline{\mathbf{u}}^{(i)}$ and $\underline{\mathbf{u}}^{(c)}$ respectively,

$$\underline{\underline{\mathbf{H}}}^{(i)} = \underline{\underline{\mathbf{Grad}_X \mathbf{u}}}^{(i)}, \underline{\underline{\mathbf{H}}}^{(c)} = \underline{\underline{\mathbf{Grad}_X \mathbf{u}}}^{(c)}, \tag{4}$$

the "complete" Green–Lagrange strain tensor, i.e. between the undeformed and compressed configurations, can be written as follows:

$$\begin{aligned}\underline{\underline{\mathbf{E}}}^{(f)} &= \frac{1}{2}\left(\underline{\underline{\mathbf{H}}}^{(f)} + \underline{\underline{\mathbf{H}}}^{(f)^T}\right) + \frac{1}{2}\underline{\underline{\mathbf{H}}}^{(f)^T}\underline{\underline{\mathbf{H}}}^{(f)} \\ &= \underline{\underline{\mathbf{E}}}^{(i)} + \underline{\underline{\mathbf{E}}}^{(c)} + \frac{1}{2}\left(\underline{\underline{\mathbf{H}}}^{(i)^T}\underline{\underline{\mathbf{H}}}^{(c)} + \underline{\underline{\mathbf{H}}}^{(c)^T}\underline{\underline{\mathbf{H}}}^{(i)}\right).\end{aligned} \tag{5}$$

Then, the strain field related to compression noted $\underline{\underline{\mathbf{E}}}^{(ci)}$ which accounts for the prestressed configuration (i) [21] can be expressed as

$$\underline{\underline{\mathbf{E}}}^{(ci)} = \underline{\underline{\mathbf{E}}}^{(f)} - \underline{\underline{\mathbf{E}}}^{(i)} = \underline{\underline{\mathbf{E}}}^{(c)} + \frac{1}{2}\left(\underline{\underline{\mathbf{H}}}^{(i)^T}\underline{\underline{\mathbf{H}}}^{(c)} + \underline{\underline{\mathbf{H}}}^{(c)^T}\underline{\underline{\mathbf{H}}}^{(i)}\right). \tag{6}$$

Fig. 2 shows the strain field on the outer liner related to compression in the form of the first and the second invariants $I_E^{(ci)}$ and $Q_{\underline{\underline{E}}}^{(ci)}$ of $\underline{\underline{\mathbf{E}}}^{(ci)}$ as well as a representation of the magnitude and the orientation of the principal strains. The first invariant $I_E{}^{(ci)}{}_{ou}$ represents the mean strain in tension or compression and the second invariant $Q_{\underline{\underline{E}}\ ou}^{(ci)}$ represents the mean shear strain [11]. These maps reveal significant compression and shear strain states along the ridges in both $\underline{\mathbf{e}}_Y$ and $\underline{\mathbf{e}}_X$ directions: the strain field is highly heterogeneous and locally exhibits high gradients. Such greatly disturbed fields are related to the particularly complex elasto-plastic behaviour of the corrugated board as well as to local structural and/or mechanical imperfections in the board or in the box. For instance, the corrugated board was intentionally damaged in such regions, i.e. creased and folded, for converting operations. Properly analysing and modelling the behaviour of box ridges and corners is out of the scope of the present study. Therefore, we have first chosen to focus our attention on the modelling of the behaviour of the box panels that are assumed without imperfections by using a elastic post-buckling model adapted to initially weakly deformed (or prestressed) plates.