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A simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates

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ABSTRACT

This paper presents a simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates. Unlike the conventional first-order shear deformation theory, the present first-order shear deformation theory contains only four unknowns and has strong similarities with the classical plate theory in many aspects such as governing equations of motion, boundary conditions, and stress resultant expressions. Equations of motion and boundary conditions are derived from Hamilton's principle. Closed-form solutions of simply supported plates are obtained and the results are compared with the exact 3D and quasi-3D solutions and those predicted by other plate theories. Comparison studies show that the present theory can achieve the same accuracy of the conventional first-order shear deformation theory which has more number of unknowns.

1. Introduction

Functionally graded materials (FGMs) are a class of composites that have continuous variation of material properties from one surface to another and thus eliminate the stress concentration found in laminated composites. A typical FGM is made from a mixture of ceramic and metal. These materials are often isotropic but nonhomogeneous. The reason for interest in FGMs is that it may be possible to create certain types of FGM structures capable of adapting to operating conditions. The increase in FGM applications requires accurate models to predict their responses. A critical review of more recent works on the static, vibration and stability analysis of functionally graded (FG) plates can be found in the paper of Jha et al. [1]. Since the shear deformation has significant effects on the responses of FG plates, shear deformation theories such as first-order shear deformation theory (FSDT) and higher-order shear deformation theories (HSDTs) should be used to analyze FG plates.

The FSDT accounts for the shear deformation effects by linear variation for in-plane displacements and requires a shear correction factor, whereas the HSDTs account for the shear deformation effects by higher-order variations for in-plane displacements or both in-plane and transverse displacements. For example, Reddy [2,3] developed a third-order shear deformation theory (TSDT) with cubic variations for in-plane displacements. Xiang et al. [4,5] proposed a n-order shear deformation theory in which Reddy's theory can be considered as a specific case. Based on the mixed variational approach, Fares et al. [6] proposed a HSDT with linear and parabolic variations for in-plane and transverse displacements, respectively. Meanwhile, the HSDTs presented by Reddy [7], Chen et al. [8], Pradyumna and Bandyopadhyay [9], and Talha and Singh [10] are developed based on cubic variations for in-plane displacements and a parabolic variation for transverse displacement. Neves et al. [11] developed a HSDT with cubic and parabolic variations for in-plane and transverse displacements, respectively, based on Carrera's unified formulation. In company with the use of polynomial functions in aforementioned works, trigonometric functions are also employed in the development of HSDTs. For example, Zenkour [12] presented a generalized shear deformation theory in which the in-plane displacements are expanded as sinusoidal types across the thickness. Mantari et al. [13–16] proposed trigonometric shear deformation theories which account for adequate distribution of the transverse shear strains across the thickness and satisfy the stress-free boundary conditions on the plate surface without using a shear correction factor. Based on Carrera's unified formulation, Ferreira et al. [17] developed a HSDT with the use of sinusoidal functions for both in-plane and transverse displacements, whereas Neves et al. [18,19] proposed HSDTs with the use of different expansions for in-plane and transverse displacement (i.e., sinusoidal [18] or hyperbolic [19] expansion for in-plane displacements and parabolic expansion for transverse displacement). Some of the abovementioned HSDTs are computational costs due to additional unknowns introduced to the theory (e.g., theories by Pradyumna and Bandyopadhyay [9] and Neves et al. [11,18,19] with nine unknowns, Reddy [7] with eleven unknowns, Talha and Singh [10] with thirteen unknowns).

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Although some well-known HSDTs have five unknowns as in the case of FSDT (e.g., the third-order shear deformation theory [2], the sinusoidal shear deformation theory [12], and the trigonometric shear deformation theories [13–15]), their equations of motion are much more complicated than those of FSDT. Thus, needs exist for the development of shear deformation theory which is simple to use.

The aim of this paper is to develop a simple FSDT for the bending and free vibration analysis of FG plates. Unlike the conventional FSDT, the present one contains only four unknowns and has strong similarities with the classical plate theory (CPT) in many aspects such as equations of motion, boundary conditions, and stress resultant expressions. The partition of the transverse displacement into the bending and shear parts leads to a reduction in the number of unknowns and governing equations, hence makes the theory simple to use. Equations of motion are derived from Hamilton's principle. Closed-form solutions of simply supported plates are obtained. Numerical examples are presented to verify the accuracy of the present theory.

2. Theoretical formulation

2.1. Kinematics

In this study, further simplifying assumptions are made to the conventional first-order shear deformation theory so that the number of unknowns is reduced. The displacement field of the conventional first-order shear deformation theory is given by

$$\begin{aligned} u_1(x,y,z) &= u(x,y) + z\varphi_x \\ u_2(x,y,z) &= v(x,y) + z\varphi_y \\ u_3(x,y,z) &= w(x,y) \end{aligned} \tag{1}$$

where u, v, w, φ_x and φ_y are five unknown displacement functions of the midplane of the plate; and h is the thickness of the plate. By deviding the transverse displacement w into bending and shear parts (i.e., $w = w_b + w_s$) and making further assumptions given by $\varphi_x = -\partial w_b/\partial x$ and $\varphi_y = -\partial w_b/\partial y$, the displacement field of the new theory can be rewritten in a simpler form as

$$\begin{aligned} u_1(x,y,z) &= u(x,y) - z\frac{\partial w_b}{\partial x} \\ u_2(x,y,z) &= v(x,y) - z\frac{\partial w_b}{\partial y} \\ u_3(x,y,z) &= w_b(x,y) + w_s(x,y) \end{aligned} \tag{2}$$

Clearly, the displacement field in Eq. (2) contains only four unknowns (u, v, w_b, w_s). In fact, the idea of partitioning the transverse displacements into the bending and shear components is first proposed by Huffington [20], later adopted by Krishna Murty [21], Senthilnathan et al. [22], Shimpi [23], and recently by Thai and his colleagues [24–41].

The nonzero strains associated with the displacement field in Eq. (2) are:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} \tag{3a}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - z\frac{\partial^2 w_b}{\partial y^2} \tag{3b}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w_b}{\partial x \partial y} \tag{3c}$$

$$\gamma_{xz} = \frac{\partial w_s}{\partial x} \tag{3d}$$

$$\gamma_{yz} = \frac{\partial w_s}{\partial y} \tag{3e}$$

2.2. Equations of motion

Hamilton's principle is used herein to derive equations of motion. The principle can be stated in an analytical form as

$$0 = \int_0^T (\delta U + \delta V - \delta K)dt \tag{4}$$

where δU, δV, and δK are the variations of strain energy, work done, and kinetic energy, respectively. The variation of strain energy is calculated by

$$\begin{aligned} \delta U &= \int_A \int_{-h/2}^{h/2} (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_{xy}\delta\gamma_{xy} + \sigma_{xz}\delta\gamma_{xz} + \sigma_{yz}\delta\gamma_{yz}) dA\, dz \\ &= \int_A \left[N_x \frac{\partial \delta u}{\partial x} - M_x \frac{\partial^2 \delta w_b}{\partial x^2} + N_y \frac{\partial \delta v}{\partial y} - M_y \frac{\partial^2 \delta w_b}{\partial y^2} + N_{xy}\left(\frac{\partial \delta u}{\partial y} + \frac{\partial \delta v}{\partial x}\right) \right. \\ &\quad \left. -2M_{xy}\frac{\partial^2 \delta w_b}{\partial x \partial y} + Q_x \frac{\partial \delta w_s}{\partial x} + Q_y \frac{\partial \delta w_s}{\partial y} \right] dA \end{aligned} \tag{5}$$

where N, M, and Q are the stress resultants defined by

$$(N_x, N_y, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) dz \tag{6a}$$

$$(M_x, M_y, M_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) z\, dz \tag{6b}$$

$$(Q_x, Q_y) = \int_{-h/2}^{h/2} (\sigma_{xz}, \sigma_{yz}) dz \tag{6c}$$

The variation of work done by external forces can be expressed as

$$\delta V = -\int_A q\delta(w_b + w_s)dA \tag{7}$$

where q is the transverse load.

The variation of kinetic energy can be written as

$$\begin{aligned} \delta K &= \int_V (\dot{u}_1 \delta \dot{u}_1 + \dot{u}_2 \delta \dot{u}_2 + \dot{u}_3 \delta \dot{u}_3)\rho(z) dA\, dz \\ &= \int_A \{ I_0[\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + (\dot{w}_b + \dot{w}_s)\delta(\dot{w}_b + \dot{w}_s)] \\ &\quad -I_1\left(\dot{u}\frac{\partial \delta \dot{w}_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial x}\delta\dot{u} + \dot{v}\frac{\partial \delta \dot{w}_b}{\partial y} + \frac{\partial \dot{w}_b}{\partial y}\delta\dot{v}\right) \\ &\quad +I_2\left(\frac{\partial \dot{w}_b}{\partial x}\frac{\partial \delta \dot{w}_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial y}\frac{\partial \delta \dot{w}_b}{\partial y}\right)\Big\} dA \end{aligned} \tag{8}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_1, I_2) are mass inertias defined by

$$(I_0, I_1, I_2) = \int_{-h/2}^{h/2} (1, z, z^2)\rho(z) dz \tag{9}$$

Substituting the expressions for δU, δV, and δK from Eqs. (5), (7), and (8) into Eq. (4) and integrating by parts, and collecting the coefficients of δu, δv, δw_b, and δw_s, the following equations of motion are obtained:

$$\delta u: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u} - I_1 \frac{\partial \ddot{w}_b}{\partial x} \tag{10a}$$

$$\delta v: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v} - I_1 \frac{\partial \ddot{w}_b}{\partial y} \tag{10b}$$

$$\begin{aligned} \delta w_b: \quad & \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q \\ & = I_0(\ddot{w}_b + \ddot{w}_s) + I_1\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) - I_2 \nabla^2 \ddot{w}_b \end{aligned} \tag{10c}$$