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An analytical study of the non-linear vibrations of functionally graded cylindrical shells subjected to thermal and axial loads

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ABSTRACT

An analytical method and a new simplifying model of FG (functionally graded) cylindrical shells are presented based on Hamilton's principle, Von Kármán non-linear theory and the first-order shear deformation theory, and subjected to thermal and axial loads. The coupled non-linear partial differential vibration equations are discretized based on a series expansion of linear modes and a multiterm Galerkin's method. Neglecting the membrane inertias and rotary inertias, the equation of motion is transformed into a reduced equation in the generalised transverse displacement. Adopting multiple scales method, the amplitude frequency dependence and the non-linear forced frequency response are obtained in the case of a single mode assumption. The good agreement found was very satisfactory, in comparison with previous theoretical approaches. The purpose of this approach was to provide engineers and designers with an easy method for determining the non-linear vibration behaviour of FG cylindrical shells.

1. Introduction

The continuous growing of the commercial use of space facilities has led to the development of new and more efficient aerospace vehicles; therefore, new and accurate studies on lightweight, thin-walled structures are needed. Many studies were concerned with cylindrical shells that constitute main parts of aircrafts, rockets, missiles and generally aerospace structures. The cylindrical shells are often forced to vibrate at large amplitudes by their environment. It is immensely useful to predict the dynamic behaviour serving as complete structures or structural components.

The study of the dynamic behaviour of shells within the scope of linear models has been the subject of numerous works [1–3]. The geometrical non-linearity generated by large vibration amplitudes causes difficulties in the analysis of the associated non-linear dynamic behaviour. The studies of large vibrations amplitudes of shells involving geometrical non-linearities require efficient non-linear procedures. Reddy [4] developed the non-linear higher-order shear deformation theory of plates, taking into account von Kármán type non-linear terms. Amabili and Reddy [5] developed a consistent higher-order shear deformation non-linear theory for shells of generic shape; geometric imperfections are also taken into account. The geometrically non-linear strain–displacement relationships are derived retaining full non-linear terms in the in-plane displacements. Amabili and Païdoussis [6] systematically studied the non-linear dynamics and large vibration amplitudes of cylindrical shells, mainly focused on non-linear free and forced vibrations of cylindrical shells. Based on the Donnel-Mushtari shell equations considering finite deformations and the Airy stress function, Sofiyev and Aksogan [7] studied the free vibrations of laminated non-homogeneous orthotropic thin cylindrical shells with geometric non-linearity. In the non-linear vibration analysis, a continuous elastic system is reduced to a discrete system with one or more degrees of freedom, and a set of non-linear ordinary differential equations in time is obtained. The system of non-linear ordinary differential equations is analysed by using the harmonic balance method, multiple scale method and Runge–Kutta method [8,9]. Based on a *p*-version, hierarchical, first-order shear deformation finite element, Ribeiro [10,11] analysed the geometrically non-linear vibrations of plates and shells under the combined effect of thermal fields and mechanical excitations. Using non-linear normal modes, Touzé et al. [12] derived reduced-order models for large amplitude, geometrically non-linear vibrations of thin shells, and compared the accuracy of different truncations, using linear and non-linear modes, in order to predict the response of shells structures subjected to harmonic excitation. To obtain the non-linear response of cylindrical shells, large numerical models are usually employed. These analyses involving a large number of degrees of freedom are very expensive with respect to both storage and CPU time. An attractive alternative is to derive consistent reduced order models that can capture the main characteristics of the shell behaviour. Many attempts have been made to develop

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suitable and simple mathematical models for predicting the non-linear behaviour of shells.

In recent years, Functionally graded materials (FGMs) have been receiving much more attention in engineering communities, especially in applications for high-temperature environment [13–17], because of the advantages of being able to withstand severe high-temperature gradient while maintaining structural integrity.

As for non-linear dynamic behaviour of FG structures, Woo et al. [18] developed the non-linear free vibration of functionally graded plates. The governing equations for thin rectangular plates of functionally graded materials are obtained using the von Kármán theory, which considers moderate deflections and small strains. Using the mesh-free kp-Ritz method, Zhao and Liew [19] developed the non-linear response of functionally graded ceramic–metal plates under mechanical and thermal loads. Yang and Huang [20] studied the sensitivity of non-linear vibration and dynamic response of FG plates to initial geometrical imperfections of arbitrary shape. Recently, Yang et al. [21] analyzed the non-linear vibration of a simply supported FG rectangular plate with a through-width surface crack. The cracked plate was treated as an assembly of two sub-plates connected by a rotational spring at the cracked section whose stiffness is calculated through stress intensity factor. By using superposition method, Galerkin method and harmonic balance method, Sofiyev [22] analyzed the non-linear vibration of an FG truncated conical shell.

In the present paper, the non-linear vibration of FG cylindrical shells subjected to thermal and axial loads is studied using the multiple scales method. The non-linear formulation is based on the first-order shear deformation theory and the von Kármán strains. The discretized equations of motion are obtained by the multiterm Galerkin's method using modal expansions for the displacement that satisfy simply supported boundary condition. Neglecting the membrane inertias, rotary inertias and the modal interaction terms, the single mode approximation is given for the amplitude frequency dependence and the non-linear forced frequency response of FG cylindrical shells. Numerical tests indicated that the simplifying model is reasonable based on comparisons between the present result and that of the previous literatures.

2. Theoretical formulations

Fig. 1 shows the FG cylindrical shell, where (x,θ,z) denote the orthogonal curvilinear coordinates such that x and θ curves are the lines of curvature on the middle surface ($z = 0$). The length, thickness and the middle surface radius of shells are denoted by L, h and R respectively.

Suppose that a typical FGM property F is varied across the shell thickness according to the power law expressions [23]

$$F(z) = F_o\left(\frac{z+h/2}{h}\right)^{\Phi} + F_i\left[1-\left(\frac{z+h/2}{h}\right)^{\Phi}\right] \quad (0 \leqslant \Phi \leqslant \infty) \tag{1}$$

where F_o and F_i denote the property of the outer $\left(z=\frac{h}{2}\right)$ and inner surface $\left(z=-\frac{h}{2}\right)$ of the shell, respectively, and Φ expresses the volume fraction exponent. Here, the elastic constants, thermal expansion coefficients, thermal conductivity and mass density are considered as varying along the shell thickness according to Eq. (1).

Form the first-order shear deformation theory, the displacements $(u_1,\ v_1,\ w_1)$ of a point (x,θ,z) in the FG cylindrical shell are expressed as sum of the midsurface displacements (u,v,w) along the x, θ and z direction, and rotations $(\varphi_x,\varphi_\theta)$ of the normals to the mid-surface along x and θ axes, as follows

$$\begin{aligned}
u_1(x,\theta,z,t) &= u(x,\theta,t) + z\varphi_x(x,\theta,t),\\
v_1(x,\theta,z,t) &= v(x,\theta,t) + z\varphi_\theta(x,\theta,t),\\
w_1(x,\theta,z,t) &= w(x,\theta,t).
\end{aligned} \tag{2}$$

The geometrically non-linear membrane strain, the transverse shear strains and curvatures are given by adding the Von Kármán terms to the linear strain displacement relations [24,25]

$$\begin{aligned}
&\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_\theta = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right) + \frac{1}{2}\left(\frac{1}{R}\frac{\partial w}{\partial \theta}\right)^2,\\
&\gamma_{x\theta} = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{1}{R}\frac{\partial w}{\partial x}\frac{\partial w}{\partial \theta}, \quad \gamma_{xz} = \varphi_x + \frac{\partial w}{\partial x}, \quad \gamma_{\theta z} = \varphi_\theta + \frac{1}{R}\frac{\partial w}{\partial \theta},\\
&\kappa_x = \frac{\partial \varphi_x}{\partial x}, \quad \kappa_\theta = \frac{1}{R}\frac{\partial \varphi_\theta}{\partial \theta}, \quad \kappa_{x\theta} = \frac{\partial \varphi_\theta}{\partial x} + \frac{1}{R}\frac{\partial \varphi_x}{\partial \theta}.
\end{aligned} \tag{3}$$

The stress–strain relation including the temperature effects is given by

$$\boldsymbol{\sigma} = \mathbf{C}(z)[\bar{\boldsymbol{\varepsilon}} - \boldsymbol{\alpha}(z)\Delta T], \tag{4}$$

where ΔT is the temperature increment referenced to the stress free state. The elasticity matrix $\mathbf{C}(z)$ and thermal expansion coefficients matrix $\boldsymbol{\alpha}(z)$ of FGM are given by Kadoli and Ganesan [26]:

$$\mathbf{C}(z) = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0\\ C_{21} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{33} & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}, \tag{5}$$

$$\boldsymbol{\alpha} = [\,\alpha_{xxe} \quad \alpha_{\theta\theta e} \quad 0 \quad 0 \quad 0\,]^T. \tag{6}$$

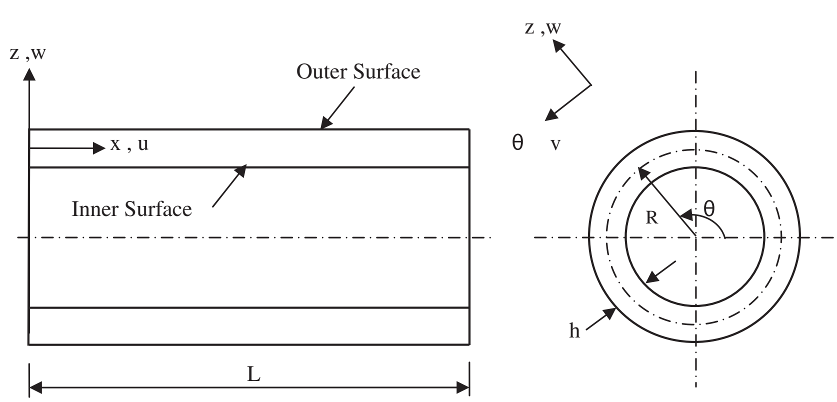

Fig. 1. Coordinate system of the functionally graded cylindrical shell.