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Perturbation-based stochastic finite element analysis of the interface defects in composites via Response Function Method

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ABSTRACT

The main aim of this paper is to present a mathematical model and its numerical realization for the composite material with stochastic interface defects employing the perturbation-based stochastic finite elements. The defects are modeled as the semicircular micro-cavities ("bubbles") lying with their diameters on the reinforcement-matrix interface inside the weaker material region (matrix). Both total number of the defects as well as their radii may be defined as the truncated Gaussian random variables with the given first two probabilistic moments. The composite micro-geometry is simulated according to these parameters and the entire structure is discretized in both scales using traditional Finite Element Method procedures. Its probabilistic version is provided thanks to the Response Function Method, where several numerical tests with random parameter values varying around its mean value enable to determine the structural response and, thanks to the Least Squares Method, its final probabilistic moments. The proposed technique has been validated on the shear test for the carbon–epoxy composite with the few interface defects and Young modulus of the matrix being Gaussian random variable, where three meshes with various density show overall convergence of the numerical procedure.

1. Introduction

Multiscale modeling of composite materials and their structural or interface defects in both deterministic fatigue fracture and certain stochastic models is still computationally attractive [8] and practically justified [9,12]. It may be based on some computer models relevant to the contact mechanics [16], some existing interface numerical models for composites [13] and, on the other hand, on various probabilistic strategies including Monte-Carlo simulation [4,10], spectral stochastic finite elements [6] and, of course, on the stochastic perturbation technique [11]. The major issue is to discover an influence of these microdefects and their parameters like geometrical dimensions, concentration ratio and shape (and particularly their statistics) on stochastic response of the composite and its reliability.

The main objective of this study is an application of the generalized Stochastic perturbation-based Finite Element Method (SFEM) to computational modeling of random interface defects in composite materials. Numerical model obeys two scales – (a) micro one connected with the Representative Volume Element of the composite and (b) geometrical scale of the interface, where these defects are located. The stochastic interface defects are defined uniquely here by two uncorrelated truncated Gaussian distributions – of their radii and total number at the interface considered – using the corresponding expected values and variances. Contrary to the previous numerical studies we use now full tenth order perturbation in derivation of the first four probabilistic moments of the composite response. Let us note that an application of the Direct Differentiation Method is unable in this model since relations in-between finite element stiffness (even for linear isotropic elasticity) and the defects parameters are unavailable, therefore the Response Function Method implemented with the least squares approximation is employed to approximate these functions through several FEM experiments. We introduce for this purpose the new parametric domain around the mean values of the random design parameters and discretize it into the few usually equidistant intervals. Their nodal values are the basis for the defects' parametrization and that is common decisive issue in many stochastic multiscale models analyzed with our version of the SFEM.

Then, the proposed approach enables to determine numerically up to the fourth central probabilistic moments and the coefficients of the structural response – stresses and deformations – as the functions of the interface defects random dispersions and expectations of their geometrical parameters, i.e. radius and total number of the cavities along the interface. Essential computational analysis is provided using the Finite Element Method with the mesh refined around the interface defects, while the entire probabilistic analysis is programmed and done after the FEM data

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transfer in the computer algebra system MAPLE. Multi-scale dependence of probabilistic characteristics for the state functions vs. interface defects parameters are demonstrated and discussed with respect to both random dispersion of the defects and stochastic perturbation analysis order. It is done with the use of symbolic computing procedures to optimize time consumption and overall effort in the numerical analysis carried out. Further simulations will be focused on a verification of the homogenization procedure [7,10] related to the interface scale to replace the defects with statistically homogeneous artificial interphase in-between original constituents of the composite considered.

2. Mathematical model of the composite

Let us consider a periodic random fiber composite in plane strain with parallel fibers in the unstressed and undeformed state. The section of this composite structure $Y \subset \Re^2$ in the plane $x_3 = 0$, orthogonal to the fiber direction is shown in Figs. 1 and 2 in the macro- and micro-scale, respectively. Let us consider the rectangular periodicity cell Ω of Y and let the geometric dimensions of Ω be related to the corresponding dimensions of Y by a certain geometrical scale parameter [10,11].

Further, let Ω contain n coherent regions, where $n \in N$, $n < \infty$ satisfying the following conditions:

$$\Omega = \bigcup_{a=1}^{n} \Omega_a \cup \bigcup_{a=2}^{n} \Gamma_{(a-1,a)}, \tag{1}$$

$$\Omega_a \cap \Omega_b = \emptyset, \quad \text{for } a \neq b, 1 \leqslant a, b \leqslant n, \tag{2}$$

where $\Gamma_{(a\text{-}1,a)}$ is a bounded and sufficiently regular contour being a real boundary between the regions Ω_{a-1} and Ω_a (two different materials). Let us consider such a class of periodicity cells Ω that for every a an interior of contour $\Gamma_{(a-2,a-1)}$ is contained in the interior of contour $\Gamma_{(a-1,a)}$ and that these contours are disjoint. Let us consider the micro-contact phenomenon between the components Ω_{a-1} and Ω_a on the interface $\Gamma_{(a-1,a)}$ (cf. Fig. 3) and let us assume the approximation of the material discontinuities occurring on this boundary by random "bubbles" (semi-circles) with both their radii $r(\omega)$ and total number at the given interface $n(\omega)$ being random; the "bubble" diameters are coinciding with the boundary $\Gamma_{(a-1,a)}$ (whose curvature is neglected and such that the 'bubbles' cannot overlap). The parameters of the bubbles are assumed to be Gaussian random variables limited to the nonnegative values only. Further let for every Ω_a the expected values and variances of the defect radii and frequency of occurrence be given. A geometrical idealization and assumptions concerning the RVE imply that random distribution of material interface defects in terms of their radii and total number are exactly the same in each cell with a single fiber.

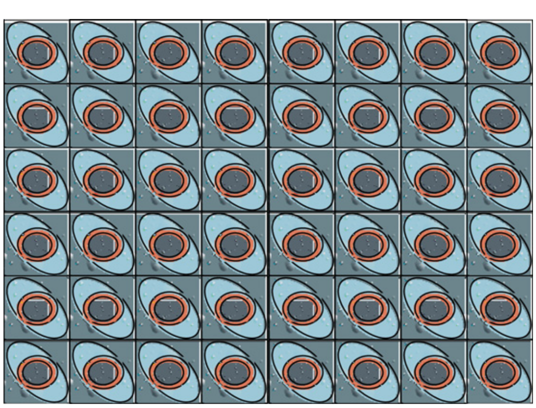

Fig. 1. The composite structure Y.

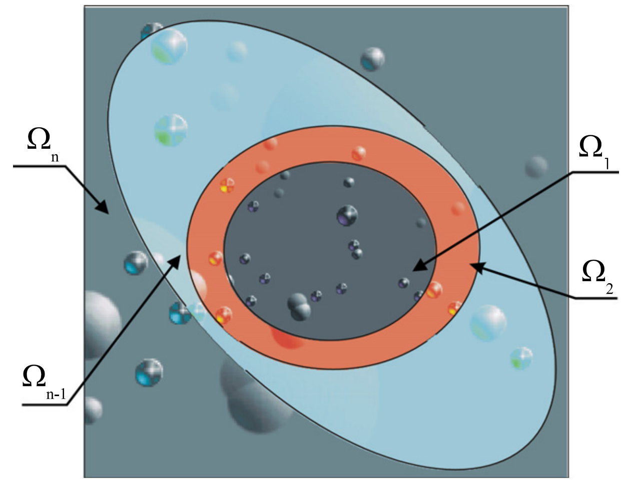

Fig. 2. The RVE of composite structure in the micro-scale.

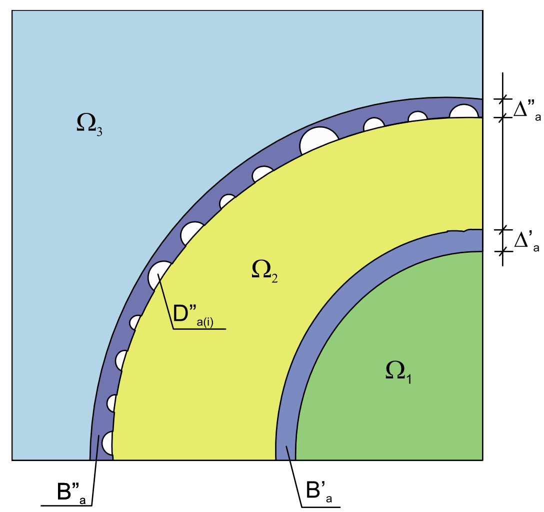

Fig. 3. The stochastic interface defects boundary in the micro-contact scale.

We assume also that the virgin and continuous materials constituting this composite are linear elastic and transversely isotropic defined with the use of the random elasticity tensor field C_{ijkl} $(\mathbf{x}\ \omega)$ is defined as follows

$$C_{ijkl}(\mathbf{x};\omega) = e(\mathbf{x};\omega) \left\{ \delta_{ij}\delta_{kl} \frac{\nu(\mathbf{x})}{(1+\nu(\mathbf{x}))(1-2\nu(\mathbf{x}))} + (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \frac{1}{2(1+\nu(\mathbf{x}))} \right\}, \tag{3}$$

where $i, j, k, l = 1, 2$; all the above specified elastic characteristics are assumed equal to 0 for any defect area $\mathbf{x} \in D_a$, $1 \leqslant a \leqslant n$. Random fields of the Young modulus and Poisson ratios are so defined that they have different expectations for various composite constituents and they are all uncorrelated variables, so that all cross-covariances equal to 0 [10].

3. Governing equations of the linear elasticity with RFM

Let us consider a statistically heterogeneous and bounded continuum $\Omega \subset \Re^2$ without no initial stresses and strains. Elastic properties and geometry of Ω may be treated as design random parameters and they result in random displacement field $u_i(\mathbf{x};\omega)$ and random stress tensor $\sigma_{ij}(\mathbf{x};\omega)$ satisfying linear elasticity