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Technical Note

Efficient prediction of the response of layered shells by a dynamic stiffness approach

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ABSTRACT

In [1] an Equivalent Single Layer (ESL) approach resulting in an efficient and accurate prediction of the response of layered flat panels was presented and validated using experimental results. The application of this approach to curved and cylindrical shells is hereby discussed. The difficulties encountered when attempting to directly apply the Wave Finite Element (WFE) homogenization procedure to curved structures are initially described. Subsequently, an approach accounting for the curvature of the panel within the formulation of the ESL is given. The results of the presented approach for a thick sandwich cylindrical shell are compared to the results of a full three-dimensional FE modelling. The accuracy and the efficiency of the approach are eventually discussed.

1. Introduction

Layered structures of various geometries are widely used in the aerospace and the automotive industries. The prediction of the vibroacoustic response of layered structures is therefore of essential importance during the design process of such industrial products. In order to accurately predict the response of multilayered panels nowadays three-dimensional FE modelling or Layer-Wise (LW) theories are typically employed. These approaches may offer results accurate for a wide frequency range, however they imply great calculation costs mainly because of the fact that the number of Degrees of Freedom (DoF) per node employed during the modelling process depends on the number of layers comprised within the structure. Thus, introducing a fine mesh to maintain interpolation and pollution errors to acceptable levels can result in prohibitive calculation costs. Equivalent Single Layer (ESL) approaches are also used in order to reduce the number of DoF to be solved, however they usually offer poor predictions regarding the higher order modes. In [1] it was shown that using a three-dimensional WFE modelling to calculate the wave propagation characteristics within layered panels can provide a basis for modelling their dynamic response.

The prediction of the vibratory response of thick layered shells has recently been a popular field of research. Among the refined shell theories a distinction has been made in [2] between theories for which the number of the unknown variables is independent or dependent on the number of the layers of the shell. The former category is usually referred to as ESL or global approaches while the latter one is referred to as LW models. Considering the ESL approaches, a concise summary of the strain displacement equations, the stress strain equations and the equations of motion for layered shell structures is given in [3–5]. The analogous of both the Kirchhoff–Love and the Mindlin type theories (usually named after Donnell–Mushtari and Flügge respectively) are presented. Higher order theories for curved structures can be found in the bibliography, however they result in increased computational effort and mathematical complexity. Ganapathi et al. [6] used a higher order theory to conduct dynamic analysis of laminated thick cylindrical shells while in [7] the authors used a higher order theory to develop closed-form solutions for the vibration of thick shells. On the other hand LW theories are usually superior to ESL approaches in terms of accuracy however they result in excessive computational cost when it comes to multilayered structures. Recently, a LW theory was used in [8] to investigate free vibrations of shell structures. In [9] the authors studied the dynamic behaviour of cylindrical layered structures with viscoelastic layers, while in [10] the authors used a LW approach to formulate shell finite elements. A well made bibliographic report on the recently developed theories of layered shells is given in [11].

The WFEM involves the coupling of Periodic Structure Theory (PST) (see [12]) to the FEM. The wave dispersion characteristics within the layered media can then be accurately predicted for a very wide frequency range, by solving a polynomial eigenvalue problem for the direction dependant propagation constants. The WFEM for two-dimensional singly curved panels and cylindrical shells has been formulated in [13] by modelling a trapezoid

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frustum segment of the original curved structure. The wavenumbers and the wave types propagating in the layered shell for each frequency range were computed. The ring frequency of the shell is also correctly predicted. In [14] the wave propagation characteristics within doubly curved panels are also computed.

The main novelty of the work hereby presented is the formulation of an ESL approach for curved thick structures of arbitrary layering. The WFEM is employed in order to compute the wave dispersion characteristics within the layered shell to be modelled. The complex intralayer shear deformation effects are therefore captured through a three-dimensional FE representation of a structural segment. The computed wave propagation characteristics are subsequently used through an homogenization process in order to update a classic shell theory and accurately predict the dynamic response of the structure. The approach is therefore capable of taking into account for the complex shear deformation effects of the structures while avoiding the complicated kinematic assumptions of higher order theories. The accuracy and the efficiency of the presented ESL approach are discussed through a numerical validation case.

The paper is formulated as follows: In Section 2 the application of the ESL approach to layered shells is discussed and a solution for overcoming the geometry posed obstacles is suggested. In Section 3 numerical examples are presented in order to validate the presented approach, while in Section 4 its efficiency compared to refined shell theories is discussed. Finally in Section 5 conclusions are given on the presented work.

2. Application of the ESL approach to a shell structure

2.1. Homogenization procedure

A singly curved thick shell of arbitrary layering and anisotropy is hereby considered (see Fig. 1). Following the analysis in [1], the homogenization procedure for curved structures would involve the computation of the wavenumbers propagating within the shell using the WFEM and subsequently the direct comparison of the computed wavenumbers to exact values for classic shell theories in order to determine the dynamic material characteristics of the ESL. Such an analysis would give accurate predictions on the purely circumferential modes of the shell, however with regard to the modelling of the stiffness effects below the ring frequency towards the axial direction the approach would encounter two major challenges:

- It is particularly difficult to encounter exact relationships between the axially propagating wavenumbers and the mechanical characteristics of the shell structure. Approximate solutions for the Donnell–Mushtari and the Flügge theories can be found in [15,16] respectively. However, the flexural/axial coupling effects within a shell imply that the flexural wavenumber cannot be expressed merely as a function of the flexural stiffness and the mass of the structure, as is the case with flat panels. The use of such wavenumber relations would therefore imply a source of approximation and a significantly increased complexity of the problem.
- Even if a 'neat' expression of the wavenumbers as a function of the mechanical characteristics of the shell proves feasible, the application of the approach would be hindered by the geometric stiffening effects. In order to avoid such effects from disturbing the accuracy of the solution the modelled ESL should present the same ring frequency as the original layered shell.

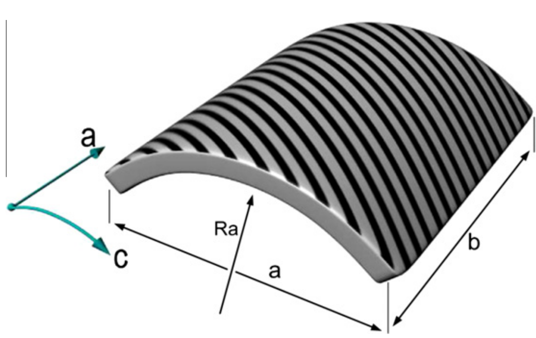

Fig. 1. A composite singly curved panel modelled within the current approach.

Using the aforementioned considerations, the calculation of the dynamic mechanical properties for the ESL is conducted in the following section.

2.2. Expressions for the dynamic characteristics

The ESL structure should now have an equal flexural stiffness and surficial density as well as a ring frequency equal to the one of the original layered shell. Considering the relations of the three aforementioned quantities as a function of the mechanical characteristics for a Donnell–Mushtari type thin shell we have:

$$f_r = \frac{1}{2\pi R}\sqrt{\frac{E_a}{\rho}}$$
$$\rho_s = \rho h \tag{1}$$
$$D_{a,c} = \frac{E_{a,c}h^3}{12(1-\nu^2)}$$

with f_r the ring frequency, $E_{a,c}$ the Young's modulus in the axial and circumferential directions, ν the Poisson's ratio, h the thickness of the ESL, ρ its density and R the radius of the shell. Considering the equivalence between the Donnell–Mushtari and the Kirchhoff–Love theories it can be deduced that the axial and circumferential flexural stiffnesses should be related to the flexural wavenumbers by the relation:

$$\widehat{D}_{a,c} = \frac{\omega^2\rho_s}{\hat{k}^4_{f,WFE}} \tag{2}$$

with $\hat{k}_{f,WFE}$ the WFEM calculated flexural wavenumbers of the flat layered panel and ^ represents the frequency dependence. Using Eqs. (1) and (2) the equivalent mechanical characteristics for the ESL can be deduced as:

$$\widehat{E}_a = \sqrt{\frac{\rho_s^3}{12\widehat{D}_a}}\omega_r^3R^3(1-\nu^2) = \frac{\rho_s\hat{k}^2_{f,WFE,a}}{\sqrt{12}\omega}\omega_r^3R^3(1-\nu^2)$$
$$\widehat{E}_c = \frac{12(1-\nu^2)\omega^2\rho_s}{\hat{k}^4_{f,WFE,c}\hat{h}^3}$$
$$\hat{\rho} = \frac{\widehat{E}_a}{\omega_r^2R^2(1-\nu^2)} = \frac{\rho_s\hat{k}^2_{f,WFE,a}}{\sqrt{12}\omega}\omega_rR \tag{3}$$
$$\hat{h} = \frac{\rho_s}{\hat{\rho}}$$

Table 1
Mechanical properties of materials.

Material I	Material II
ρ = 1500 kg/m^3	ρ = 58 kg/m^3
E_x = 57 GPa	E_x = 78 MPa
E_y = 53 GPa	E_y = 78 MPa
ν_{xy} = 0.10	ν_{xy} = 0.20
G_{xy} = 3.45 GPa	G_{xy} = 8.7 MPa
–	G_{yz} = 42.6 MPa
–	G_{xz} = 38.3 MPa