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A statistical failure initiation model for honeycomb materials

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ABSTRACT

In the current study, a statistical failure initiation model for honeycomb materials is proposed. The model describes the failure initiation in macroscopic normal-shear stress space. The workflow of the model is explained in three stages: micromechanical model, simulation experiments under external macro-stresses and boundary conditions, and analysis of the experiment results. In the micromechanical model, the heterogeneous nature of the material is described with the geometrical parameters which are irregularity and scale and their variations. Based on these parameters, samples are designed and simulation experiments are conducted. The experiment results are linked to possible failure mechanisms in order to obtain the critical macroscopic stresses which are expressed in terms of cumulative distribution functions. Further investigations on these functions with the weakest link theory and Weibull distribution lead to understand and quantify the effects of the geometrical parameters on the failure initiation characteristic in a statistical manner. As a result of these investigations, the statistical model describing the failure initiation of honeycomb materials is presented as functions of the irregularity and scale in macroscopic stress space.

1. Introduction

Honeycomb materials are widely used for various structural applications due to the minimized amount of material usage and high stiffness-to-weight ratios [1]. They can easily function as tailored solutions for different special problems due to their design as interconnected network of solid struts. Since being a popular material in engineering applications, there have been studies related to their deformation and failure mechanisms in the literature. These include the research activities on the elastic properties of regular honeycombs for which the struts form equilateral cells with exactly the same corner angles [2–4]. Such analyses are not, however, feasible since they do not take the micro-structural variations into account [5]. This necessitated developing the models for irregular honeycombs of which there is random deviation in shape and size of their cell structures [6,7]. Although the effects of irregularities and imperfections on the elastic and failure properties have been wisely modelled, the results have been mostly analysed in deterministic manner [8]. However, taking heterogeneous nature of the honeycomb materials into consideration, more realistic results can be obtained by blending these micromechanical models with statistical analysis techniques [9].

In order to complete this missing link in the literature, the proposed model, which is an extension of the conference articles [10,11], aims at expressing the effects of geometrical parameters such as irregularity and scale on the failure initiation of the honeycomb materials in a compact, statistical form. The present investigations start with a micromechanical model for honeycomb materials which explains the geometrical and mechanical relations separately. In this micromechanical model, the heterogeneous nature of the material is described with the geometrical parameters such as irregularity and scale and their variations. Based on these parameters, samples are designed and simulation experiments are conducted. The experiment results are linked to possible failure mechanisms to obtain the critical macroscopic stresses and analyze the effects of irregularity and scale in terms of statistical methods. The outcome of the analysis is the generic cumulative distribution function in macroscopic stress space describing the failure initiation probabilities of the honeycomb materials.

2. Overview

2.1. Objectives

The main objective of the present study is to develop a statistical failure initiation model for honeycomb materials in macroscopic stress space. The model is designed so that once the failure initiation statistics of a reference honeycomb sample is known for a specific macro-stress combination, it is possible to predict the failure initiation statistics for honeycombs of various geometrical features and scales under the same macro-stresses.

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For a convenient representation, the failure initiation statistics is described in terms of cumulative distribution function which gives the probability with which a material fails under the critical macro-stress value. As a result of this study, the critical macro-stress limits and failure initiation probabilities of the honeycomb materials can be obtained and catastrophe can be avoided well in advance.

2.2. Statistical failure initiation model

In order to form the statistical failure initiation model, experiments are carried out under various external macro-stress combinations $\underline{\mathbf{s}}$ for both regular honeycombs composed of equilateral cells with same corner angles and irregular honeycombs with variations in cell sizes and corner angles. Thereafter, the experiment results are linked to possible failure mechanism(s) and the critical macro-stress values $\mathbf{s}_{cr}$ are calculated and plotted in a chosen stress space, e.g. macroscopic normal-shear stress space, as illustrated in Fig. 1. Since $\mathbf{s}_{cr}$ is unique for regular honeycombs under each stress combination and is independent of size of the material, the outcome is always a curve defining the maximum possible safe region. If this curve is exceeded, the failure initiation is unavoidable. In case of irregular honeycomb(s) of irregularity α and scale $\underline{V}$, the phenomenon is explained with a variation domain instead of a curve. As shown in Fig. 1, the variation domain can be described in terms of cumulative distribution function which gives the probability with which a material fails under $\mathbf{s}_{cr}$. By using necessary statistical tools and cumulative distributions, it is possible to describe the variation domain for wide range of α and $\underline{V}$.

2.3. Methodology

In order to describe the variation domain and understand the effects of the parameters on the failure initiation, samples of various α and $\underline{V}$ should be tested either physically or virtually. However, it is a very challenging task to carry out physical experiments due to the need for excessive amount of data. Therefore, the present study focuses only on simulation experiments. For this purpose, a micromechanical model is developed as explained in Fig. 2. By means of this model, both microscopic and macroscopic mechanical field quantities are calculated under different $\underline{\mathbf{s}}$ and boundary conditions. Then, the failure mechanism(s) to be used is decided, e.g. bending moment can be taken to be decisive for failure due to its dominance on cell wall deformation of the honeycombs. In the following step, the critical bending moment M_{cr} and the critical macro-stress $\mathbf{s}_{cr}$ causing the first beam to fail are linked to each other in terms of maximization and scaling operations. Then, $\mathbf{s}_{cr}$ values for honeycombs of different α and $\underline{V}$ are analyzed by using statistical tools such as the weakest link and Weibull theories. The outcome of the analysis is the generic cumulative distribution function which describes the failure initiation probabilities of the honeycomb materials.

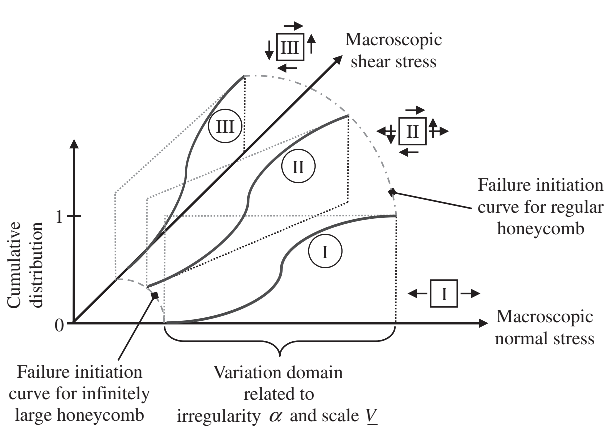

Fig. 1. Statistical failure initiation model for honeycomb material.

3. Micromechanical model

The current micromechanical model represents the honeycomb material as a heterogeneous medium where the errors related to homogenization and discretization can be eliminated by direct calculations for the microscopic mechanical quantities [12]. The following subsections explain this modeling approach in detail.

3.1. Material element

The material element is a $H \times H$ square honeycomb as seen in Fig. 3. The independent geometrical variables are the measure of geometrical irregularity α and the dimensionless scale parameter $\underline{V} = (H/h^0)^2$, where H is the specimen size and h^0 is the cell wall height for the regular honeycomb material. The measure of irregularity α is the ratio between the cell wall height offset Δh and h^0 as shown in Fig. 3. Here, the regularity of the material is described by a one-parameter model, in which $\alpha = 0$ and $\alpha > 0$ correspond to regular and irregular material geometries, respectively. To be more precise, the cell vertices are given random offsets of magnitude $\Delta h = \alpha h^0$ and the uniform distribution interval is determined as $[h^0 - \Delta h, h^0 + \Delta h]$. However, the scale $\underline{V} = (H/h^0)^2$ is treated in a deterministic way, while the shapes and sizes of the cells, cell wall elastic modulus E_s and thickness t of the cell walls are assumed to be known.

In the laboratory XY-coordinate system, the domain occupied by the specimen is $\Omega = [0,H] \times [0,H]$, where $\partial\Omega$ represents the boundary domain.

3.2. Beam equations

In the micromechanical model, the cell walls of Fig. 3 are modeled as elastic Bernoulli beams. The material inside the cells is taken to be soft compared to that of the walls. In this beam model, the equilibrium equations in beam xyz-Cartesian coordinate system are

$$\vec{F}' + \underline{\vec{f}} = 0, \tag{1}$$

$$\vec{M}' + \vec{i} \times \vec{F} + \underline{\vec{m}} = 0, \tag{2}$$

in which unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ are codirectional with x-, y-, and z-axes. The prime symbol is used to denote the derivative with respect to x. In Eqs. (1) and (2), $\underline{\vec{f}}$ and $\underline{\vec{m}}$ are the external load parameters, and $\vec{F}$ and $\vec{M}$ are the stress resultants acting on a beam cross-section. The constitutive equations for $\vec{F}$ and $\vec{M}$ of a linearly elastic material take the forms

$$\vec{F} = (E_s A u_x')\vec{i} - (E_s I u_y''')\vec{j}, \tag{3}$$

$$\vec{M} = (E_s I u_y'')\vec{k}, \tag{4}$$

in which u_x, u_y are the displacement components in the directions of the x- and y-axes. Elastic modulus E_s is the constant material parameter of the problem. The geometrical properties of beam cross-sections are $A = \mathcal{T}t$ and $I = \mathcal{T}t^3/12$ in which t is the cell wall thickness (same as the beam depth) and $\mathcal{T}$ is the thickness of the honeycomb material (same as the beam width). The x-axis of the xyz-coordinate system is assumed to coincide with the neutral axis of the beam.