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Free vibration of cantilevered composite plates in air and in water

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ABSTRACT

Composite materials are being used more frequently for marine applications due to the advantages of a higher stiffness- and strength-to-weight ratio, and better corrosion resistance compared to metallic alloys. Many examples consist of cantilevered structures, such as hydrofoils, propeller and turbine blades, keels, and rudders. A wide range of analytical and numerical tools exist for the free vibration analysis of composite structures in air due to their applicability to design problems in the aerospace industry, such as airplane wings, turbofan and propeller blades, and flight control surfaces. For these aerospace structures the inertial effects of the fluid are typically neglected due to the low relative density of air compared to the structure. Contrarily, for marine structures, fluid inertial (added mass) effects cannot be neglected, especially for composites with much higher fluid-to-solid density ratios. The objective of this work is to investigate the effects of material anisotropy and added mass on the free vibration response of rectangular, cantilevered composite plates/beams via combined analytical and numerical modeling. The results show that the natural frequencies of the composite plate are 50–70% lower in water than in air due to large added mass effects. The added mass is found to vary considerably with material orientation due to the bend-twist coupling of anisotropic composites, which affects the mode shapes and, consequently, the fluid inertial loads. The analytical method is found to yield accurate results for beam geometries and offers significant savings in computational cost compared to the finite element method.

1. Introduction

The use of composite materials for marine applications is becoming more prevalent due to the advantages of a generally higher strength- and stiffness-to-weight ratio, and better corrosion resistance compared to traditional metallic alloys such as steel, aluminum, or bronze. Additionally, composite materials offer the ability to elastically tailor the deformation through the design of the material (e.g. via the laminate stacking sequence), which can yield better performance over a wider range of operating conditions [9,10,12,16–18,21,27].

Many marine applications for which composite materials have been used consist of cantilevered structures, including propeller and turbine blades, hydrofoils, keels, and rudders. One important aspect of the design of these structures is free vibration analysis, where the natural frequencies and mode shapes are calculated in order to predict the dynamic structural response, as well as to identify potential instability limits such as resonance and flutter. Similar design problems may be found in the aerospace industry, including aircraft wings, turbofan and propeller blades, helicopter rotors, and wind turbines, and many of these have benefitted greatly from the use of composite materials. Consequently, many analytical and numerical tools have been developed and validated for composite structures in air.

Narita and Leissa [19] presented an analytical method for calculating the dry natural frequencies and mode shapes of very thin composite plates using the Ritz method. They analyzed a large number of configurations, varying both the aspect ratio and the laminate layup sequence, and the results were shown to compare favorably with both numerical (finite-element) and experimental results of Crawley [6]. Analytical methods have also been developed for Timoshenko beams [4,22] for varying geometries and laminate layup sequences. In each of these studies, the effects of the fluid inertia, i.e. added mass, of the surrounding fluid are neglected due to the relatively low density of air compared to the structural density. This assumption, however, is not typically valid for marine structures, for which added mass effects are often significant. Consequently, methods for the prediction of the free vibration of composite structures must be developed with consideration for fluid inertial effects.

The effect of added mass on the natural frequencies increases as the ratio of fluid-to-solid density increases, and consequently the added mass of most structures in water may not be neglected, especially for composite structures, which generally have a lower

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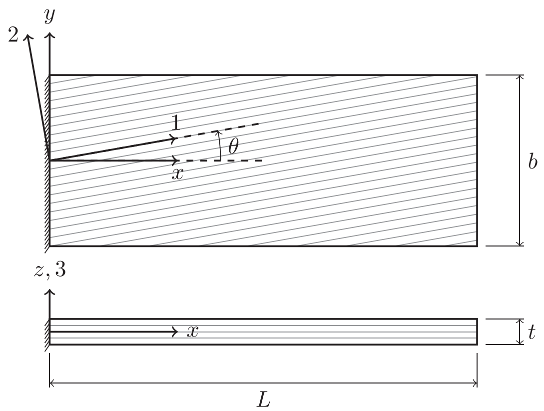

Fig. 1. Diagram of a typical rectangular composite cantilevered plate or beam. Aspect ratio is defined as $\mathcal{R} = L/b$, thickness-to-chord ratio is represented as t/b.

effective density than metallic structures. The added mass effects for cantilevered isotropic plates have been investigated by several authors, including Marcus [14] and Yadykin et al. [26], to name a few. In [14], a finite element method is used, where the fluid effects are modeled using inviscid, linearly compressible acoustic fluid elements. The results were found to compare well with the experimental results of Lindholm et al. [11]. In [26], an analytical formulation is presented for a submerged, cantilevered, isotropic plate in water, where the fluid inertial effects are calculated based on potential flow theory. Although several methods for considering fluid inertial effects have been developed, they have previously been applied to isotropic structures constructed of metallic alloys. Since, in general, added mass effects are highly dependent on the nature of the vibrational modes (e.g. bending or twisting), the bend-twist coupling that results from the material anisotropy present in composite materials introduces more complex added mass effects that must be quantified.

The objective of this work is to investigate the effects of material anisotropy and added mass on the free vibration response of rectangular, cantilevered composite plates/beams via combined analytical and numerical studies. An analytical method is first presented, followed by numerical modeling via finite element simulations. Convergence and validation studies are shown. The influence of added mass and material anisotropy on the free-vibration response of composite plates and beams are examined, followed by a summary of the major findings.

2. Problem description

The current study is aimed towards cantilevered structures such as wings, hydrofoils, propeller and turbine blades, keels, and rudders. Due to the variability in the geometry of these structures, they are generalized in this study as rectangular, cantilevered plates and beams for simplicity. It is expected that the major phenomena exhibited by the rectangular plates/beams are representative of those present for more complicated structures.

In this work, two fluid media are considered: air and water, denoted as dry and wet configurations, respectively. For the dry cases, the air is assumed to have a negligible effect and is ignored (i.e. fluid density, $\rho_f = 0$). A schematic for a typical cantilevered plate or beam is shown in Fig. 1. The dimensions are defined by its length, width, and thickness, represented by L, b, and t, respectively. The aspect ratio is defined as $\mathcal{R} = L/b$, which is in line with its definition in aerodynamics, and the thickness-to-chord ratio is defined as t/b. Results for two different geometries will be shown in this paper: one that is representative of a thin plate, with $\mathcal{R} = 2.6$ and $t/b = 3.46\%$, and one that satisfies beam assumptions, with $\mathcal{R} = 12$ and $t/b = 20\%$. The dimensions for each geometry are shown in Table 1. It should be noted that the length of both the plate and the beam, as well as the material properties, are identical, and that only the aspect ratio and thickness ratio are varied.[1]

Table 1
Dimensions of the plate and beam considered in this study.

Dimension	Plate	Beam
L	243.8 mm	243.8 mm
b	92.50 mm	20.32 mm
t	3.200 mm	4.063 mm
$\mathcal{R} = L/b$	2.6	12
t/b	3.46%	20%

A right-handed coordinate system is placed with the x–y plane at the mid-plane of the plate, where the origin is located at the cantilevered end and the z-direction is in the thickness direction (see Fig. 1). The principle axes of the composite material are denoted as the 1–2 axes, where the 1-direction is along the effective fiber angle, which is oriented at an angle θ with respect to the x–y axes.

In general, a composite plate/beam is composed of multiple laminate plies, and each ply may be aligned at a different angle. It has been shown that, for symmetric layup sequences, an equivalent unidirectional fiber angle may be found to represent the overall load-deformation characteristics, and hence simplify the coupled fluid–structure interaction (FSI) analysis [17,28]. Although the internal stress distributions of the structure with the equivalent unidirectional fiber angle will not match with the actual multi-layered composite structure, the deformation behavior, and consequently the natural modes and frequencies, will be equivalent. In this paper, the equivalent fiber angle is denoted as θ and is used as a means for parametric studies to explore effect of material anisotropy more generally. It should be noted that both the analytical and numerical methods presented in this paper are capable of handling generalized composite layups.

The material is modeled as orthotropic, where the 1-direction is associated with the fiber direction, and the stiffness in the 2- and 3-directions is assumed to be equivalent. Under this assumption, the entire material behavior may be specified by five properties, E_1, E_2, G_{12}, ν_{12}, and ν_{23}, where E_i is the Young's modulus in the i-direction and G_{ij} and ν_{ij} are the shear modulus and Poisson's ratio in the i–j plane, respectively. Symmetry considerations require the following: $E_2 = E_3$, $\nu_{12} = \nu_{13}$, $G_{12} = G_{13}$ and $G_{23} = E_2/2(1 + \nu_{23})$. The assumed material properties for the composite plate/beam are shown in Table 2.

3. Analytical formulation

The analytical solution for the free vibration of composite beams with consideration for FSI is presented in this section. The model accounts for the coupling between bending and torsion induced by material anisotropy. The beam's center of gravity is assumed to be collocated with the elastic axis to enable independent study of material coupling effects. In line with Bernoulli–Euler beam theory, the current model neglects shear deformation, rotary inertia, and warping effects. The added mass formulas used in the analytical model are based on potential flow assumptions and strip theory.

Each longitudinal section is restricted to two degrees of freedom: z-displacement and x-rotation about the reference axis, denoted as h and ϕ, respectively, as shown in Fig. 2. The governing partial differential equations for such a structural system under

[1] Additional geometries and material configurations are examined in Section 4.4 for validation studies with published results from the literature. However, all original results presented in this paper are for one or both of these two geometries.