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Hygrothermal postbuckling behavior of functionally graded plates

Chang-Yull Lee [a], Ji-Hwan Kim [b,*]

[a] School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, South Korea
[b] Institute of Advanced Aerospace Technology, School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, South Korea

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ABSTRACT

This work considers the postbuckling behaviors of Functionally Graded Material (FGM) plate in hygrothermal environments. Basically, the structures change continuously in the thickness direction according to the volume fractions of the material. For the description of the model, the first-order shear deformation theory (FSDT) is used, and von Karman strain–displacement relations are applied. In the analysis, finite element method and Newton–Raphson technique are adopted to analyze the thermal postbuckling behavior of the model. Furthermore, a simple power-law is employed in the thickness direction of the plate, and the temperature and moisture effects are fully investigated in this study. To check the validity of the present work, comparisons with the previous results are performed. And then, moisture effects on the model are significantly appeared due to the increase of the volume fraction index of the materials.

1. Introduction

Functionally Graded Materials (FGMs) have been used in the severe thermal loading conditions. Then, thermal buckling and dynamic instability phenomena et al. are occurred frequently. Therefore, the high temperature environment may be a critical factor of the structures. On the other hand, moisture absorption may be practically occurred during the operation of aerospace, marine and submarine structures et al. Then, the sub-structure may be involved in serious reduction of the stiffness and strength. Thus, the hygrothermal effects on the mechanical system have been a hot issue in the various fields.

The effects were investigated for laminated composite structures at first. In the theoretical point of view, Whitney and Ashton [1] researched on bending, buckling and vibration of plates. Gandhi et al. [2] considered nonlinear vibration behaviors of structures. Shen [3] investigated the influence of the environments on the postbuckling behavior of higher-order plate models. Yang et al. [4] performed a transient analysis for single-layered cylinder with the coupling of heat and moisture effects. Benkhedda et al. [5] proposed an analytical approach for the stresses in the plate model and the change of mechanical characteristics due the simultaneous variation of moisture and temperature. Also, Lo et al. [6] investigated based on the global–local higher order plate theory, and proposed a method on the concentration effects of the temperature and moisture. Patel et al. [7] studied the static and dynamic characteristics of structures using first order and high order theory. Also, Zenkour [8] presented the bending analysis results for FGM plate resting on elastic foundations. Also, Lee and Kim [9] performed hygrothermal behaviors of FGM.

On the other hand, the concepts of Functionally Graded Materials (FGMs) have been introduced to overcome drawbacks of the composite materials [10], and also the continuous change of microstructure is well known as the distinguished feature. Yang and Shen [11] investigated the nonlinear bending behavior of the panel for the increasing temperature. Using 3-D finite element model, Na and Kim [12] studied the volume fraction optimization by considering stress and the critical temperature. Sohn and Kim [13] performed the thermal postbuckling and nonlinear flutter of the panel in a supersonic flow. Singha et al. [14] suggested the modified shear correction factors for the FGMs. Further, Magada and Jaroslaw [15] analyzed the vibrations of the functionally graded macrostructure using tolerance averaging technique. On the other hand, Kapuria et al. [16] validated static and free vibration response of the beam model using experimental results.

In this paper, thermal postbuckling behavior of the FGM considering hygrothermal as well as moisture effects are analyzed. The material is assumed to follow a simple power-law of mixture. The models are based on the first-order shear deformation plate theory and von Karman strain–displacement relations are adopted in the structural modeling. Newton–Raphson iterative method is applied to solve matrix equations, and then numerical results are compared with the previous data. Further, the more discussions are summarized for the degree of the moisture concentration and volume fraction index.

2. Formulations of FGMs

Fig. 1 shows the FGM model with length a, width b and thickness h, respectively.

* Corresponding author. Address: 599 Kwanak-ro, Kwanak-ku, Seoul 151-744, South Korea. Fax: +82 2 887 2662

E-mail address: jwhkim@snu.ac.kr (J.-H. Kim).

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2.1. FGM properties

The material properties are assumed to be made up of a mixture of ceramic and metal. And the properties are varied continuously as well as smoothly in the thickness direction. Further, a simple power-law distribution is adopted, thus the volume fractions of the ceramic V_c and metal V_m are expressed as in Ref. [17].

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^k \quad (0 \leqslant k < \infty), \quad V_m(z) = 1 - V_c(z) \tag{1}$$

where V, the superscript k, the subscripts c and m represent the volume fraction, the volume fraction index, ceramic and metal, respectively.

Using the rule of mixture, then

$$\begin{aligned} P_{eff}(T,z) &= P_m(T)V_m(z) + P_c(T)V_c(z) \\ &= P_m(T) + (P_c(T) - P_m(T))\left(\frac{z}{h} + \frac{1}{2}\right)^k \end{aligned} \tag{2}$$

where P_{eff}, P_m and P_c in here P is the property of material and subscript $_{eff}$ stands for effective property.

Further, the properties of FGM at high temperature conditions can be expressed as shown in Table 1:

$$P(T) = P_0\left(\frac{P_{-1}}{T} + 1 + P_1T + P_2T^2 + P_3T^3\right) \tag{3}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients of temperature [18].

2.2. Constitutive equations

Basically, the first-order shear deformation theory of the plate is employed:

$$\begin{aligned} &u_\alpha(x,y,z,t) = u_\alpha^0(x,y,t) + z\phi_\alpha(x,y,t) \ (\alpha = x,y) \\ &w(x,y,z,t) = w_0(x,y,t) \end{aligned} \tag{4}$$

where $u_\alpha^0(x,y,t)$ and w_0 are the mid-plane displacements in the α and z directions, respectively. Additionally ϕ_x and ϕ_y are the rotations of the normal to the xz and yz planes, respectively.

In this study, the von Karman strain–displacement relations are adopted, thus the in-plane strain vectors are

$$\mathbf{e}^T = \{u_{,x}, v_{,y}, u_{,y} + v_{,x}\} + \frac{1}{2}\{w_{,x}^2, w_{,y}^2, 2w_{,x}w_{,y}\} \tag{5}$$

Using these two equations, the strain–displacement relationship in terms of in-plane strains with subscript 0 and curvature strains in brace are

$$\begin{aligned} \mathbf{e}^T = &\{u_{0,x}, v_{0,y}, u_{0,y} + v_{0,x}\} + \frac{1}{2}\{w_{0,x}^2, w_{0,y}^2, 2w_{0,x}w_{0,y}\} \\ &+ z\{\phi_{x,x}, \phi_{y,y}, \phi_{x,y} + \phi_{y,x}\} \end{aligned} \tag{6}$$

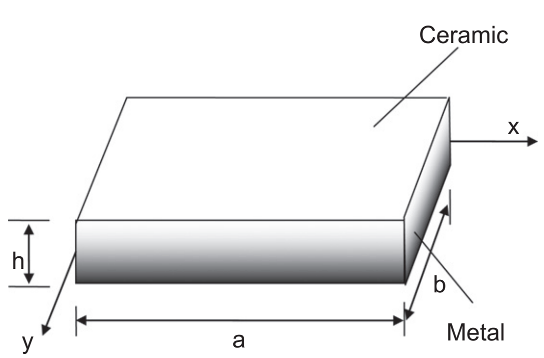

Fig. 1. Geometry of a functionally graded plate.

Further, transverse shear strains are

$$\gamma^T = \{w_{0,y} + \phi_y, w_{0,x} + \phi_x\}^T \tag{7}$$

On the other hand, the stress–strain relations for the hygrothermal condition as in Ref. [8] are

$$\begin{aligned} &\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \end{Bmatrix} = \frac{E}{1 - v^2}\begin{bmatrix} 1 & v \\ v & 1 \end{bmatrix}\begin{Bmatrix} \varepsilon_{xx} - \alpha\Delta T - \beta\Delta C \\ \varepsilon_{yy} - \alpha\Delta T - \beta\Delta C \end{Bmatrix} \\ &\{\sigma_{yz}, \sigma_{xz}, \sigma_{xy}\} = \frac{E}{2(1+v)}\{\varepsilon_{yz}, \varepsilon_{xz}, \varepsilon_{xy}\} \end{aligned} \tag{8}$$

where $\Delta T = T - T_0$ and $\Delta C = C - C_0$ in here T_0 and C_0 are reference temperature and the reference moisture concentration, respectively. Also, α and β are thermal and moisture expansion coefficients, respectively.

Then, the constitutive equations for the plates in hygrothermal environment are

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}\begin{Bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \end{Bmatrix} - \begin{Bmatrix} \mathbf{N}_{\Delta\mathbf{T}} \\ \mathbf{M}_{\Delta\mathbf{T}} \end{Bmatrix} - \begin{Bmatrix} \mathbf{N}_{\Delta C} \\ \mathbf{M}_{\Delta C} \end{Bmatrix}, \quad \mathbf{Q} = \mathbf{A_s}\gamma \tag{9}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$, $\mathbf{Q}$ and $\mathbf{A_s}$ are matrixes representing the extensional, bending-extension coupling, bending, the resultant shear force and shear stiffness matrices, respectively. Moreover, the matrices are defined as $(\mathbf{A},\mathbf{B},\mathbf{D}) = \int_{-h/2}^{h/2}\mathbf{E}(1,z,z^2)dz$ and $\mathbf{A_s} = \kappa_p\int_{-h/2}^{h/2}\frac{E(z)}{2(1+v)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}dz$. In here, $E(z)$ is the z- dependent Young's modulus of the FGM in the thickness direction, and κ_p is the shear correction factor including the effect of mixture of materials. For FGM plate, material properties vary in the thickness direction, thus a constant shear correction factor is not appropriate [19]. Therefore, the factor is replaced as $\kappa_p = \frac{5}{6-(v_mV_m+v_cV_c)}$, where v_m and v_c stand for the Poisson ratios of metal and ceramic, respectively according to the Ref. [20]. Additionally, $(\mathbf{N}_{\Delta\mathbf{T}},\mathbf{M}_{\Delta\mathbf{T}})$ and $(\mathbf{N}_{\Delta C},\mathbf{M}_{\Delta C})$ are defined as the temperature and moisture dependent quantities. And the mathematical expressions are $\int_{-h/2}^{h/2}(1,z)\mathbf{E}\{\alpha(z),\alpha(z),0\}^T\Delta T(z)dz$ and $\int_{-h/2}^{h/2}(1,z)\mathbf{E}\{\beta(z),\beta(z),0\}^T\Delta C(z)dz$.

2.3. Governing equations

The principle of virtual work is used to derive the equations of motion

$$\delta W = \delta W_{\text{int}} - \delta W_{ext} = 0 \tag{10}$$

where δW_{int} and δW_{ext} represent the internal virtual work and the external virtual work, respectively.

Then,

$$\begin{aligned} \delta W_{\text{int}} &= \int_V \delta\mathbf{e}^\mathbf{T}\boldsymbol{\sigma}dV = \int_A \left[\delta\boldsymbol{\varepsilon}^\mathbf{T}\mathbf{N} + \delta\boldsymbol{\kappa}^\mathbf{T}\mathbf{M} + \delta\boldsymbol{\gamma}^\mathbf{T}\mathbf{Q}\right]dA \\ &= \boldsymbol{\delta}\mathbf{d}^\mathbf{T}\left[\mathbf{K} - \mathbf{K}_{\Delta\mathbf{H}} + \frac{1}{2}\mathbf{N1} + \frac{1}{3}\mathbf{N2}\right]\mathbf{d} - \boldsymbol{\delta}\mathbf{d}^\mathbf{T}\mathbf{P}_{\Delta\mathbf{H}} \end{aligned} \tag{11}$$

In Eq. (11), $\mathbf{d} = [\mathbf{u}, \mathbf{v}, \mathbf{w}, \boldsymbol{\varphi}_\mathbf{x}, \boldsymbol{\varphi}_\mathbf{y}]^T$ is the displacement vector, and $\mathbf{K}$, $\mathbf{K}_{\Delta\mathbf{H}}$, $\mathbf{N1}$ and $\mathbf{N2}$ denote the linear elastic, hygrothermal, first-order nonlinear and second-order nonlinear stiffness matrices, respectively. Further, $\mathbf{K}_{\Delta\mathbf{H}}$ stands for the summation of stiffness matrices $\mathbf{K}_{\Delta\mathbf{T}}$ and $\mathbf{K}_{\Delta\mathbf{C}}$, while $\mathbf{P}_{\Delta\mathbf{H}}$ represents the hygrothermal load vector.

On the other hand, the external virtual work is

$$\begin{aligned} \delta W_{ext} = \int_A &[-I_0(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w) - I_1(\ddot{\phi}_x\delta u + \ddot{u}\delta\phi_x + \ddot{\phi}_y\delta v + \ddot{v}\delta\phi_y) \\ &- I_2(\ddot{\phi}_x\delta\phi_x + \ddot{\phi}_y\delta\phi_y)]dA = -\{\delta d\}^T\mathbf{M}\ddot{\mathbf{d}} \end{aligned} \tag{12}$$

where (I_0, I_1, I_2) is obtained by $\int_{-h/2}^{h/2}\rho(1,z,z^2)dz$, and $\mathbf{M}$ is the mass matrix.