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A displacement-based finite element formulation for the analysis of laminated composite plates

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ABSTRACT

This paper presents the nodally integrated plate element (NIPE) formulation for the analysis of laminated composite plates based on the first-order shear deformation theory. The nodally integrated approach aims at providing smoothed derivative quantities by constructing nodal strain–displacement operators. Within this framework a new family of elements for plates with general monoclinic layers is developed: the strain–displacement operators are derived via nodal integration for linear triangles and quadrilateral elements. The degrees of freedom are only the primitive variables: displacements and rotations at the nodes. The NIPEs are locking-free elements, exhibit little sensitivity to geometric distortions and can be readily implemented into existing finite element codes. The efficiency of the proposed variational formulation is proved whereas effectiveness and convergence of the proposed finite elements are confirmed through several numerical applications. Finally, numerical results are compared with the corresponding analytical solutions as well as to other finite-element solutions.

1. Introduction

Laminated composite plates and shells are attractive structural components for engineers due to their extreme design flexibility. The lamination scheme and material properties of individual lamina provide further degrees of freedom to help designers tailoring the stiffness and strength of the structure to match the performance requirements. The laminated plate theories can be divided into the following two categories: (i) equivalent single layer (ESL) theories and (ii) the continuum-based 3D elasticity theory.

Generally the structural theories used to characterize the behavior of composite laminates fall into the so called equivalent single layer theories where an equivalent hypothetical single layer is defined combining the material properties of constituent layers. The ESL has been found to be adequate in predicting global response characteristics of laminates, such as maximum deflections, maximum stresses, fundamental frequencies, and critical buckling loads.

ESL includes many theories such as the classical lamination theory, the first-order shear deformation theory, the higher-order shear deformation theories, and the layer-wise lamination theory (see for example [1–7]). Among these theories, the first-order shear deformation theory (FSDT) is considered the most attractive approach owing to its simplicity, low computational cost and good compromise between numerical accuracy and computational burden.

In this context the finite element method (FEM) is especially versatile and efficient for the analysis of complex structural behavior of the composite laminated structures. The major problem is how to eliminate shear locking as the thickness-length ratio of the plate becomes small. Many numerical techniques have been proposed to overcome this phenomenon with varying degree of success. A brief review is here presented. Plate elements based on enhanced incompatible mode, see Ref. [8], with improved in-plane deformation posses excellent performance. A very effective approach is the mixed interpolation method [9–12], in which the displacement fields and the shear strain fields are interpolated independently. The approach is suitable to produce refined elements by referring to the framework introduced by the Carrera Unified Formulation (CUF) [13–15]. Within this technique the variable kinematic modeling admits linear, parabolic, cubic and fourth-order displacement fields in the thickness direction of the plate. Both equivalent single layer and layer-wise variable descriptions can be considered. In Ref. [16] a hybrid stress formulation for laminated composite plates is presented by generalizing the approach presented in [17–20] for single layer isotropic plates. A mesh-free model based on the moving Kriging interpolation is presented in Ref. [21] and employed for computing the vibration

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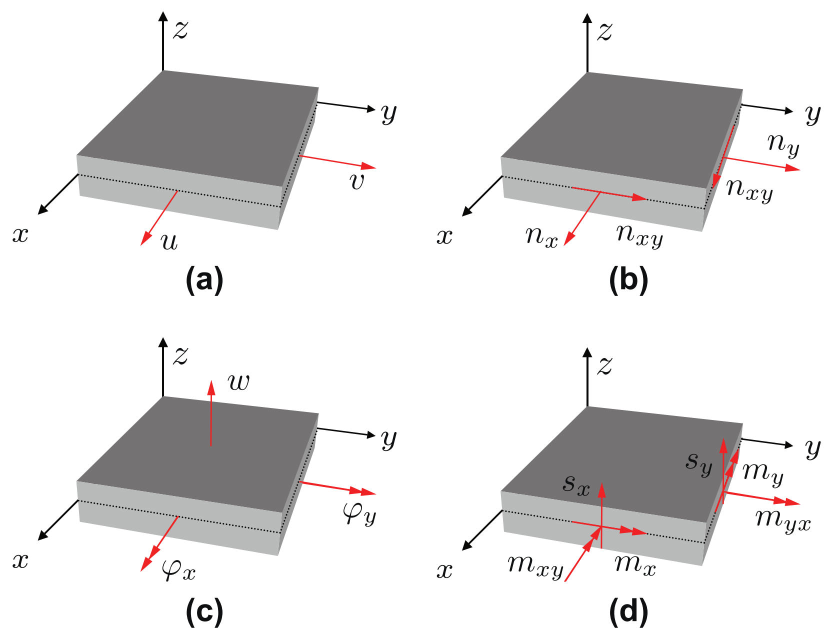

Fig. 1. Reference geometry with the indication of positive direction.

frequencies of composite laminates. The method shows acceptable accuracy and the desirable convergence rate for the free vibration analysis of laminated composite plates. Many other techniques have been proposed over the years. An interested reader could refer for instance to the literature review presented in Ref. [22].

In this work the nodally integrated plate element (NIPE) formulation is presented for the analysis of laminated composite plates based on the first-order shear deformation theory. The nodally integrated approach aims at providing smoothed derivative quantities by constructing nodal strain–displacement operators [23–26]. In particular the NIPE approach [27] develops assumed strain finite element for shear deformable plates weakly enforcing the balance and the kinematic equations. Within this framework a new family of elements for plates with general monoclinic layers is developed: the strain–displacement operators are derived via nodal integration, for linear triangles and quadrilateral elements. The degrees of freedom are only the primitive variables: displacements and rotations at the nodes. The NIPEs are locking-free elements, and exhibit little sensitivity to geometric distortions [27,28]. Finally, they are readily implementable into existing finite element codes.

1.1. Overview

The paper is organized as follows. The governing equations and the finite element formulations are stated in Section 2. In Section 3, the weighted residual formulation is derived and the formulation of the corresponding assumed-strain operator is presented in Section 4. Finally in Section 5, the performance of the procedure is assessed on some benchmark problems for laminated structures. Examples are also provided to illustrate the element performance and sensitivity to shape distortion. In Section 6, some concluding remarks are given.

2. Governing equations and basic notation of a flat shell element

Consider a plane shell element referred to the following Cartesian coordinate frame:

$$V = \{(x,y,z) \in \mathbb{R}^3 | z \in [-t/2; t/2],\ (x,y) \in A \in \mathbb{R}^2\}, \tag{1}$$

where A and t are the area and the thickness of the plane shell element respectively. The boundary of the flat shell element is $C = \partial A$. The three-dimensional displacement is denoted by $\boldsymbol{u}$ and its Cartesian components are

$$u_x = u + z\varphi_y, \quad u_y = v - z\varphi_x, \quad u_z = w, \tag{2}$$

where $\varphi_x = \varphi_x(x, y)$ and $\varphi_y = \varphi_y(x, y)$ are the rotations of the transverse normal about the Cartesian axes x and y respectively, $u = u(x,y)$ and $v = v(x,y)$ are the in-plane displacements and $w = w(x,y)$ is the deflection. The functions φ_x, φ_y, u, v, w are the unknown fields in the laminated plate bending problem (see Fig. 1), and we can conveniently express the three-dimensional displacement vector in terms of the mixed-component vector of the generalized displacements $\tilde{\boldsymbol{u}}$

$$[\tilde{\boldsymbol{u}}] = [u, v, w, \varphi_x, \varphi_y]^{\mathrm{T}}, \tag{3}$$

as

$$\boldsymbol{u} = \boldsymbol{S}\tilde{\boldsymbol{u}}, \tag{4}$$

where we introduce the shifter

$$\boldsymbol{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & z \\ 0 & 1 & 0 & -z & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{5}$$

The membrane gradient operator $\boldsymbol{D}^m$ is defined as

$$\boldsymbol{D}^m = \begin{bmatrix} \partial/\partial x & 0 & 0 & 0 & 0 \\ 0 & \partial/\partial y & 0 & 0 & 0 \\ \partial/\partial x & \partial/\partial y & 0 & 0 & 0 \end{bmatrix}, \tag{6}$$

and is used to compute membrane strains

$$\boldsymbol{\eta}^m = \boldsymbol{D}^m\tilde{\boldsymbol{u}}, \tag{7}$$

The bending gradient operator $\boldsymbol{D}^b$ is defined as

$$\boldsymbol{D}^b = \begin{bmatrix} 0 & 0 & 0 & 0 & \partial/\partial x \\ 0 & 0 & 0 & -\partial/\partial y & 0 \\ 0 & 0 & 0 & -\partial/\partial x & \partial/\partial y \end{bmatrix}, \tag{8}$$