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General higher-order shear deformation theories for the free vibration analysis of completely doubly-curved laminated shells and panels

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ABSTRACT

The main aim of this paper is to provide a general framework for the formulation and the dynamic analysis computations of moderately thick laminated doubly-curved shells and panels. A 2D higher-order shear deformation theory is also proposed and the differential geometry is used to define the arbitrary shape of the middle surface of shells and panels with different curvatures.

A generalized nine-parameter displacement field suitable to represent in a unified form most of the kinematical hypothesis presented in literature has been introduced.

Formal comparison among various theories have been performed in order to show the differences between the well-known First-order Shear Deformation Theory (FSDT) and several Higher-order Shear Deformation Theories (HSDTs).

The 2D free vibration shell problems have been solved numerically using the Generalized Differential Quadrature (GDQ) technique. The GDQ rule has been also used to perform the numerical evaluation of the derivatives of the quantities involved by the differential geometry to completely describe the reference surfaces of doubly-curved shell structures.

Numerical investigations concerning four types of shell structures have been carried out. GDQ results are compared with those presented in literature and the ones obtained using commercial programs such as Abaqus. Very good agreement is observed.

1. Introduction

As it is well known, shell structures have a great capacity in carrying external loads. This is mainly due to the curvature effect operating in shells. A large variety of classical and modern approaches for plates and shells provides appropriate tools for solving structural engineering problems. However, the increasing use of laminated shell structures in engineering applications requires refined instruments in order to achieve a deeper knowledge involving the conceiving, modeling and computing of the shell structures under investigation.

Over the years, a substantial progress has been also made in the philosophical approach to structural design. Fundamentally, the design process regards the finding and detailing of the most economical structure consistent with the safety, serviceability and aesthetic considerations. In other words, the purpose of design is to ensure that the structure being conceived will not become unfit for the use required. In design, some points have to be taken into consideration such as variations in materials of the structure, variations in loading and accuracy of design calculations. With reference to the above requirements, in this paper the type and the shape of structures, the lamination scheme and the accuracy of calculation will be involved.

The development of researches concerning the behavior of engineering structures such as plates and shells is well documented in a number of books [1–16]. A variety of solutions of important shell problems are shown in these books, which describe the foundation of various theories on which the solutions are based. Recently, a unified approach to the theoretical and applied aspects of shell analysis is reported in the book by Carrera et al. [17], where basic principles, advanced models, classical plates and shell theories, finite element applications, refined and advanced theories for evaluating static and dynamic structural responses are offered with a number of example problems. In the last decades, a significant number of Higher-order Shear Deformation Theories (HSDTs) for composite plates and shells has been published. Many researchers have furnished several results in the study of laminated composite plates and shells [18–24] by using first order, or higher order theories. For the sake of conciseness, only three of these papers will be briefly described.

Carrera [18] dealt with theories and finite elements for multilayered plates and curved shells. A unified description of several modelings based on displacements and transverse stress assumptions was given. The order of the expansion in the thickness direction

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was assumed as a free parameter. Two-dimensional modelings which include zig-zag effect, interlaminar continuity as well as layer-wise and equivalent single layer description were addressed. More than fifty theories and finite elements were compared to those developed in the framework of the unified notation.

Asadi and Qatu [22] showed equilibrium equations and the associated boundary conditions for curved deep and thick composite shells. Two First-order Shear Deformation Theories (FSDTs) were used and the curvature effect was also taken into account. The governing differential equations of the system were solved numerically via Generalized Differential Quadrature (GDQ) method. Numerical applications refer to isotropic, cross-ply-angle and lay-up cylindrical shells. Qatu et al. [24] carefully examined the dynamic behavior of laminate composite shells of different geometries between the years 2000–2009. The review article [24] was conducted with emphasis on the type of testing or analysis performed, complicating effects in material and structure, as well as on the various shell geometries. A general discussion on the various theories was also given. The review article under discussion collates the research performed in the area of dynamic analyses of composite shell structures for a period of ten years and contains 199 references.

As far as the numerical methods used for solving differential equations are concerned, the GDQ method will be used in this paper to discretize the derivatives in the governing equations. A lot of papers have been appeared over the years [25–61], and it is nearly impossible to cite all of them. The GDQ method has been applied extensively, since it gives very accurate results by using merely a few grid points. Here, only two works will be described, but all the references under discussion have evaluated the efficiency and accuracy of the GDQ method. The excellent book by Shu [25] presents the fundamentals of the Differential Quadrature (DQ) method, its properties and the error estimates. It also shows the application of the DQ method in vibration analysis of beams, plates and shells. Recently, Viola et al. [61] conducted a numerical investigation of functionally graded cylindrical shells and panels using the Generalized Unconstraint Third-order Theory (GUTSDT) coupled with the stress recovery via GDQ method. The suggested theoretical model was derived from a 2-D third-order shear deformation theory. It allows the application of distributed loads of various nature at the top and bottom surfaces of the shell. In other words, the proposed formulation does not enforce any boundary condition on the shell surfaces and maintains the unconstrained nature proper of the pioneer shear deformation theory by Timoshenko [1]. Moreover, the GUTSDT leads to accurate determination of transverse stress profiles by using 3D elasticity equilibrium equations.

Regarding the classification of plate and shell deformation theories reported in previous studies, there are mainly three major theories which are usually known as: the Classical Plate Theory (CPT) [1–11], the First-order Shear Deformation Theory (FSDT) [12–16] and the Higher-order Shear Deformation Theory (HSDT) [17]. In investigating the structural behavior of plate and shell structures, both under static and dynamic mechanical loading, as well as under thermal actions, a significant contribute has been introduced by higher order shear deformation theories. Usually, these theories remove the shear correction factor and improve the accuracy of transverse shear stresses. In fact, they approximately account for parabolic distribution of shear stresses through the thickness of the shell. It should be noted that the recent theories under discussion depend on the assumed displacement field, in which the displacements of the middle surface are expanded. As a matter of fact, the specific kinematical field assumption is suitable to characterize the theory under consideration. The above kinematical models, which have been furnished and developed to improve the analysis of shell responses, involve a lot of researchers [62–103]. Surveys of various shear deformation theories for plates and shells can be found in the paper by Mantari et al. [94], where the shear functions developed by a number of researchers are also reported in chronological order. Mantari et al. [90] developed a new shear deformation theory for laminate composite sandwich plate and shell structures. Following procedures similar to the one presented by Reddy and Liu [69], which satisfies tangential stress-free boundary conditions at the top and bottom surfaces of the plate, a new displacement field was presented. This displacement field involves five unknown displacements functions of the middle surface of the shell, as well as an unknown shear function determining the distribution of the transverse shear strains and stresses along the thickness. The assumption in [90] leads to the calculation of the maximum center plate deflection which is well compared to the one obtained using 3D elasticity bending solutions. Mantari and Guedes Soares [92] presented a generalized 5 degrees of freedom (DOF) Higher-order Shear Deformation Theory (HSDT) for plates and shells, which is suitable to reproduce many non-polynomial HSDTs available in the literature. The governing equations are solved via Navier-type closed-form solutions. Static and dynamic results are shown for cylindrical and spherical shells for simple supported boundary conditions. Numerical results refer to thick and thin shells as well as shallow and deep ones. Shear deformation theories have been also investigated by Ferreira et al. [79,87] and Neves et al. [93,103], among others. In [79] a trigonometric shear deformation theory was used for the first time for modeling symmetric composite plates discretized by a meshless method, based on global multiquadratic radial basis functions. This trigonometric theory uses trigonometric functions through the thickness direction. In [87] an analysis of laminated orthotropic elastic shells by a sinusoidal shear deformation theory and radial basis function collocation, accounting for through-the-thickness deformation, was performed. The equations of motion and the boundary conditions were obtained according to the Carrera's Unified Formulation [18]. In [93] an original hyperbolic sine shear deformation theory for the bending and free vibration analysis of functionally graded plates was presented. The effect of thickness stretching was also addressed. A different expansion for the in-plane and out-of-plane displacement was proposed. In [103] the free vibration analysis of functionally graded shells by an higher-order shear deformation theory and radial basis function collocation is carried out. The paper deals only with shells having constant curvature radius.

To the best knowledge of the authors, the literature background on the dynamic analysis of moderately thick laminated doubly-curved shells and panels is quite poor. It should be noted that some doubly-curved shells investigated in literature refer to shallow shells, as far as numerical applications are concerned. This paper covers the dynamic analysis of anisotropic and multilayered shells and panels with different curvatures, namely with arbitrary geometry, by using a displacement field having a fixed nine degrees of freedom (9DOF). The starting point of the present general higher-order formulation is the proposal of a kinematical assumption which is suitable to represent most of the kinematical hypothesis presented in literature until now. Furthermore, four new shear functions are proposed. The paper is arranged as follows. In Section 2, the theoretical framework for the general higher-order shear deformation theories concerning doubly-curved moderately thick shell structures is presented. In Section 3, the differential geometry is used to describe the middle surface of shell structures by means of the position vector written in the global reference system. The Differential Quadrature implementation is shown in Section 4, where all the typical parameters involved by the differential geometry are written in their discrete forms. In Section 5 a lot of numerical results are reported. They concern with four types of structures: a conical shell (singly-curved), an elliptic paraboloid