Composite Structures 99 (2013) 76–87

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions

Mohammed Sobhy *

Department of Mathematics, Faculty of Science, Kafrelsheik University, Kafr El-Sheikh 33516, Egypt

ARTICLE INFO

Article history:
Available online 3 December 2012

Keywords:
Exponentially graded material
Various boundary conditions
Shear deformation plate theories
Free vibration
Buckling loads

ABSTRACT

This paper deals with the vibration and buckling behavior of exponentially graded material (EGM) sandwich plate resting on elastic foundations under various boundary conditions. New functions for midplane displacements are suggested to satisfy the different boundary conditions. The elastic foundation is modeled as Pasternak's type which can be either isotropic or orthotropic and as a special case it converges to Winkler's foundation if the shear layer is neglected. The present EGM sandwich plate is assumed to be made of a fully ceramic core layer sandwiched by metal/ceramic EGM coat. The governing equations of the dynamic response of non-homogeneous composite plates are deduced by using various shear deformation plate theories. Numerical results for the natural frequencies and critical buckling loads of several types of symmetric EGM sandwich plates are presented. The validity of the present solution is demonstrated by comparison with solutions available in the literature. The influences of the inhomogeneity parameter, aspect ratio, thickness ratio and the foundation parameters on the natural frequencies and critical buckling loads are investigated.

1. Introduction

The conventional sandwich structures are generally fabricated form three homogeneous layers, two face sheets adhesively bonded to the core. However, the sudden change in material properties across the interface between different materials can result in large interlaminar stresses. To overcome these adverse effects, a new class of advanced inhomogeneous composite materials, that compose of two or more phases with different material properties and continuously varying composition distribution (using a simple functional law or an exponential law), has been developed which is referred to as functionally graded materials (FGMs). Such materials were introduced as to take advantage of the desired material properties of each constituent material without interface problems. The sandwich plate faces are typically made from a mixture of two materials. While the core of this sandwich plate is fully homogeneous material.

Studies related to FGM sandwich structures are few in numbers. Zenkour [1] was the first to introduce the sandwich structures with functionally graded faces. He studied the mechanical bending response, buckling and free vibration of simply supported FGM sandwich plate in that paper. Zenkour and Sobhy [2] investigated the thermal buckling of various types of FGM sandwich plate using sinusoidal shear deformation plate theory (SPT). An investigation of bending response of a simply supported FGM viscoelastic sandwich beam with elastic core resting on Pasternak's elastic foundations was presented by Zenkour et al. [3]. Three-dimensional finite element simulations for analyzing low velocity impact behavior of sandwich panels with a functionally graded core were conducted by Etemadi et al. [4]. Anderson [5] presented an analytical three-dimensional elasticity solution method for a sandwich composite with a functionally graded core subjected to transverse loading by a rigid spherical indentor. An exact thermoelasticity solution for a two-dimensional sandwich structures with functionally graded coating was presented by Shodja et al. [6]. In Bhangale and Ganesan [7], the buckling and vibration of a FGM sandwich beam having viscoelastic layer was studied in thermal environment by using a finite element formulation.

Composite structures on elastic foundations have wide applications in modern engineering and pose great technical problems in structural design. Winkler's elastic foundation model, which consists of infinitely many closed-spaced linear springs, is a one-parameter model that is extensively used in practice. A number of papers have dealt with vibration and buckling of plates on Winkler's foundation [8–10]. The limitation of this model is that it assumes no interaction between the springs. To overcome this problem, several two-parameter models have been suggested, such as Filonenko-Borodich, Pasternak and Vlasov and Leontev foundations, can be used [11]. Buckling loads, free vibrations and

* Tel.: +20 1091904073.
E-mail addresses: msobhy@sci.kfs.edu.eg, msobhy2011@gmail.com

0263-8223/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruct.2012.11.018

vibrations with initial in-plane stresses for moderately thick, simply supported rectangular laminates resting on elastic foundations (Pasternak's type) were examined by Aiello and Ombres [12]. Lal et al. [13] investigated the free vibration analysis of laminated composite plates resting on elastic foundation undergoing large amplitude oscillation with random system properties. Malekzadeh and Karami [14] employed the first-order shear deformation theory to study the free vibration of thick plates of continuously varying thickness on two-parameter elastic foundations using a differential quadrature solution. Free vibration analysis of vertical rectangular Mindlin plates resting on Pasternak's elastic foundations and fully or partially in contact with fluid on their one side was investigated, for different combinations of boundary conditions, by Hashemi et al. [15]. Chen et al. [16] studied the bending and free vibration of arbitrarily thick beams resting on Pasternak's elastic foundations.

The most commonly used plate theory is the classical plate theory. However, it neglects transverse shear strains and underpredicts deflections and overpredicts natural frequencies and buckling loads. In order to obtain accurate predictions of the global response characteristics and adequately describe the motion of plate-type structures, the first-order shear deformation plate theory (FPT) is established by Reissner [17]. This theory does not satisfy the stress-free boundary conditions on the surfaces of the plate and requires an arbitrary shear correction factor. To overcome these drawbacks, Reddy [18] proposed a third-order shear deformation plate theory (TPT). Touratier [19] and Zenkour [1,20] chose transverse strain distribution as a sine function. This theory may be called trigonometric or sinusoidal shear deformation plate theory (SPT). Hyperbolic shear deformation plate theory (HPT) was proposed by Soldatos [21]. Finally, Karama et al. [22] suggested an exponential variation (EPT) to investigate the effect of the transverse shear deformation on the bending of composite beams.

Several studies have been performed to analyze the behavior of composite structures using the above various shear deformation theories. The mechanical and thermal buckling analysis of functionally graded ceramic metal plates was presented by Zhao et al. [23] using the FPT. Reddy [24] analyzed the static behavior of FG rectangular plates based on his third-order shear deformation plate theory. Zenkour et al. [25–27] employed the SPT to explain the bending and thermal buckling behavior for various structures resting on two-parameter elastic foundations. Free vibrations of cross-ply laminated shells subjected to different sets of edge boundary conditions have been investigated by Timarci and Soldatos [28] on the basis of the HPT. In Akavci and Tanrikulu [29], two hyperbolic displacement models have been used for the buckling and free vibration analyzes of simply supported orthotropic laminated composite plates. Aydogdu and Taskin [30] employed the EPT to discuss the free vibration analysis of simply supported FG beam.

In the present paper, free vibrations and critical buckling loads for various types of EGM sandwich plates are investigated. The sandwich plate is assumed to be resting on isotropic or orthotropic two-parameter elastic foundations. Material properties of the sandwich plate faces are assumed to vary in the thickness direction only according to a new exponential law distribution in terms of the volume fractions of the constituents. The core layer is still homogeneous and made of an isotropic material. The governing equations of an EGM sandwich plate are given based on the sinusoidal shear deformation plate theory. The results obtained as per SPT are compared with those obtained as per the FPT, TPT, EPT and HPT. Several kinds of symmetric sandwich plates are presented. Equilibrium equations of EGM sandwich plates include the interaction between the plate and the foundations. The influences of several parameters are discussed.

2. Sandwich structures

An isotropic EGM sandwich plate of constant thickness of h with cross-sectional dimensions a and b is considered and shown in Fig. 1. The EGM sandwich plate is defined in the (x, y, z) coordinate system with x- and y-axes located in the middle plane $(z = 0)$ and its origin is placed at the corner of the plate. The external bounding planes of the sandwich plate are defined by $z = \pm h/2$. The vertical positions of the two interfaces between the core and faces layers are denoted, respectively, by h_1 and h_2. The plate is assumed to attach to the foundation so that no separation takes place in the process of deformation. The load–displacement relation between the plate and the supporting foundation is as follows [26,27]:

$$P = Kw_0 - G_x \frac{\partial^2 w_0}{\partial x^2} - G_y \frac{\partial^2 w_0}{\partial y^2}, \tag{1}$$

where P is the density of the reaction force of elastic foundation, K is the modulus of subgrade reaction (springs stiffness) and G_x, G_y are the shear moduli of the subgrade (shear layer foundation stiffness). If the foundation is homogeneous and isotropic, we will get $G_x = G_y = G$. If the shear layer foundation stiffness is neglected, the Pasternak's foundation becomes the Winkler's foundation.

The sandwich plate is made of three layers. Its faces are made of an EGM with material properties varying smoothly in the z direction only. The EGM are composed from a mixture of metal and ceramic while the core is fully ceramic. We assume that the composition is varied from the interfaces to the bottom and top surfaces, i.e. the bottom $(z = -h/2)$ and top $(z = +h/2)$ surfaces of the plate are metal-rich whereas the interfaces (h_1, h_2) are ceramic-rich. The volume fraction of the sandwich plate faces varies according to a simple power law function of z while that of the core equals unity, and they are given as:

$$\begin{aligned} V^{(1)} &= \left(\frac{2\bar{z}+1}{2\hbar_1+1}\right)^k, \quad -1/2 \leqslant \bar{z} \leqslant \hbar_1, \\ V^{(2)} &= 1, \quad \hbar_1 \leqslant \bar{z} \leqslant \hbar_2, \\ V^{(3)} &= \left(\frac{2\bar{z}-1}{2\hbar_2-1}\right)^k, \quad \hbar_2 \leqslant \bar{z} \leqslant 1/2, \end{aligned} \tag{2}$$

where $\bar{z} = z/h, \hbar_i = h_i/h \, (i = 1, 2)$ and k is the inhomogeneity parameter which takes values greater than or equal to zero. It is noted that the core is independent of the value of k which is fully ceramic. The value of k equaling to zero represents a homogeneous isotropic ceramic plate and the value of it equaling to infinity represents a metal–ceramic–metal (m–c–m) sandwich plate. The above power law assumption reflects a simple rule of mixtures used to obtain the effective properties of the metal–ceramic sandwich plate (see Fig. 1).

The mechanical properties of FGMs are often being represented in the exponentially graded form [31] and power law variations one [1–3]. Based on a new exponential law distribution, the

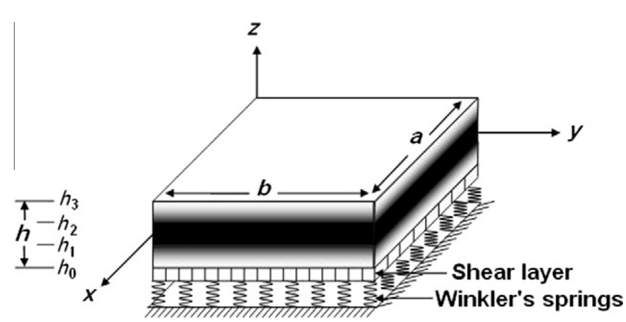

Fig. 1. Geometry of the EGM sandwich plate resting on elastic foundations.