Composite Structures 99 (2013) 141–151

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Continuum damage modeling of composite laminated plates using higher order theory

A.K. Gupta [a], B.P. Patel [a,*], Y. Nath [b]

[a] Department of Applied Mechanics, Indian Institute of Technology Delhi, Hauz Khas, New Delhi 110 016, India
[b] Department of Mechanical and Industrial Engg., Faculty of Applied Science & Engg., University of Toronto, Toronto, Ontario, Canada

ARTICLE INFO

Article history:
Available online 19 December 2012

Keywords:
Higher-order
Continuum damage
Laminated composite
Failure load
3D FEM
Shear correction factor

ABSTRACT

The aim of the paper is to check the accuracy of global plate models in predicting progressive damage and failure load of cross-ply laminated composite plates. The damage evolution model is based on a generalized macroscopic continuum theory within the framework of irreversible thermodynamics. The progressive damage analysis is carried out using global higher order shear deformation theory with/without thickness stretch and zig–zag terms, and first order shear deformation theory with shear correction factor of 5/6 and the one calculated accounting for layers' properties and lamination scheme. For comparison, progressive damage modeling employing 3-D finite element is also done. The parametric study is carried out to assess the accuracy of higher-order model based on continuum damage mechanics for different span-to-thickness ratio, lamination scheme and boundary conditions. It is concluded that higher-order model with zig–zag terms in in-plane displacement and thickness stretch terms in transverse displacement predicts failure load and stress distributions quite accurately. The higher order model is computationally efficient since the number of unknowns is independent of number of layers. For two-layered thick laminates, first order model with shear correction factor calculated based on lamination scheme predicts significantly better results compared to that with shear correction factor of 5/6.

1. Introduction

The process of progressive damage in fiber reinforced composites, due to nucleation and growth of defects such as matrix cracks, fiber breakage, fiber matrix debonding and inter-layer delamination, may cause the degradation of material properties and failure. Accurate predictions of failure initiation, progression and failure load are essential for designing reliable, safe and failure proof structures. The progressive failure analysis (PFA) can be performed by combining failure criteria, damage mechanics and fracture mechanics approaches to account for damage initiation, progression/macro-crack initiation and macro-crack propagation to a critical size. Failure criterion refers to set of mathematical equations that predict the initiation of failure by comparing the current states of stress/strain with strength properties of material [1–4]. The limitation of failure criteria approach is that the damage growth cannot be predicted. The damage progression (continuous stiffness degradation) at micro-scale and macro-crack initiation can be predicted using continuum damage mechanics [5,6].

Continuum damage mechanics (CDM) is a phenomenological approach that represents the macroscopic effects of microscopic cracks through an internal state variable (damage variable). Damage variable is derived from physically based evolution law within the framework of thermodynamics and continuum mechanics. Kachanov [7] first introduced the concept of continuum factor in his study of fracture of material under creep. To formulate the constitutive relation of damaged material, concept of effective stress [8], strain equivalence [9] and strain energy equivalence [10] principles have been introduced. Matzenmiller et al. [11] established the damage variable evolution law using the concepts of thermodynamics of irreversible processes and proposed an anisotropic damage model for fiber-reinforced composites based on in-plane failure modes, i.e. fiber failure, matrix failure and fiber–matrix shear failure. Loading surface is derived from plane stress version of Hashin criteria [2] in terms of damage energy release rates.

A number of researchers have used quadratic damage potential functions of damage energy release rate [12–21] and damage evolution is derived from extremum conditions of damage dissipation power density. The damage potential is defined as a function of damage driving forces, overall hardening parameter which defines the damage threshold, and damage tolerance parameters [12].

* Corresponding author. Tel.: +91 011 26591232; fax: +91 011 26581119.
E-mail address: badripatel@hotmail.com (B.P. Patel).

0263-8223/$ - see front matter
http://dx.doi.org/10.1016/j.compstruct.2012.11.038

Kattan and Voyiadjis [13,16], Voyiadjis and Kattan [14,15] used damage potential function coupled with micromechanics using two damage variables for matrix and fiber, separately. The components of anisotropic damage tolerance parameters were determined experimentally based on uniaxial tensile tests along three principal axes of the material [13–16] whereas Barbero and De Vivo [17], Barbero and Lonetti [18,19] derived orthotropic damage tolerance parameters from Tsai-Wu failure criterion [1] in stress space. The interlaminar damage effect is also included by Lonetti et al. [20]. The damage is characterized completely at the lamina level requiring a more extensive database of lamina strength values [17–20]. Voyiadjis and Kattan [21,22] used the concept of fabric tensors within the framework of classical elasticity theory to model the evolution of micro-crack orientation and distribution. Voyiadjis et al. [23] combined the micromechanical approach and concept of fabric tensors to evaluate the local damage of the matrix and fiber.

Ladeveze and LeDantec [24] assumed damage evolution law as linear function of equivalent damage energy release rate and proposed a mesomechanical damage model for single-ply laminate considering matrix microcracking and fiber/matrix debonding represented by two internal state variables. Allix and Ladeveze [25], Allix et al. [26], Daudeville and Ladeveze [27] extended the approach [24] to predict the interface delamination by introducing an interface layer between the laminae and three damage variables corresponding to degradation of three interface stiffness constants. These damage variables are assumed to evolve according to a power law function of equivalent damage energy release rate. Ladeveze [28] and Ladeveze et al. [29] combined the ply and interface damage models to predict the overall damage of laminate under quasi-static and dynamic loadings.

Numerical treatment of CDM based progressive damage models to determine effect of stiffness degradation on response of composite laminates under different loading conditions has been reported by various researchers. Based on classical plate theory, the effect of damage on non-linear dynamic response of laminated plates with piezoelectric layers is studied by Tian et al. [30,31] considering elasto-plastic deformation, damage evolution law similar to plastic flow rule. Tian et al. [30,31] used the stress tensor as the conjugate force to damage which is thermodynamically inconsistent [32]. Ghosh and Sinha [33] and Ghosh [34] implemented the uniaxial version of damage model of Matzenmiller et al. [11] with first-order shear deformation based finite element model to study the effect of damage on impact response of laminated composite plates and spherical panels.

Robbins et al. [35,36] implemented continuum damage model [17] in the finite element framework based on first order [35] and layerwise [36] shear deformation theories to investigate the global failure load (GFL) and effect of stiffness degradation on static response of thick laminated composite plates. It is predicted that the global failure load calculated using first-order shear deformation theory is greater than that obtained from layerwise theory. This difference may be attributed to the inaccurate prediction of interlaminar shear stress from first-order shear deformation theory for thick laminates. To accurately model the damage progression with less computational effort, Robbins and Reddy [37] used layerwise theory with adaptive kinematics to restrict the use of discrete layer kinematics to only those regions that have significant interlaminar shear effect. In the above cited studies [33–37], the effect of geometric nonlinearity is not considered which may significantly influence the damage distribution and failure load for thin structures. Gupta et al. [38,39] investigated the effect of evolving damage on static response characteristics of laminated composite shallow cylindrical/conical panels [38] and plates [39] subjected to uniformly distributed loading with inclusion of geometric nonlinearity based on the first order shear deformation theory.

The progressive damage modeling and global failure load prediction of thick composite structures is strongly affected by interlaminar shear stresses and through the thickness material inhomogeneity. However, this cannot be adequately captured by classical and first-order shear deformation theories. Therefore, the use of layerwise or higher-order theories is essential in order to accurately predict the through-thickness displacement, strain, stress fields and the effect of material degradation. In layer wise models, the number of unknowns increases with the increase in the number of layers making it computationally expensive. The layerwise models, wherein interface continuity conditions of transverse shear stresses is enforced, pose problems in determining the model coefficients as damage variables approach to unity. The main aim of the present work is to investigate the efficacy of a global higher-order displacement model including zig–zag function, which is piecewise linear with values of -1 and 1 alternately at the different interfaces [40,41], for predicting the damage evolution and failure load of thick laminated plates. The zig–zag function takes care of inclusion of the slope discontinuity of in-plane displacements at the interfaces of the laminate as observed in exact three dimensional elasticity solutions of the thick laminates. Numerical implementation of continuum damage model in to a three dimensional finite element model is also carried-out to assess the accuracy of two-dimensional models for damage progression and global failure load.

2. Continuum damage mechanics model

2.1. Constitutive equations with damage

The relation between the Cauchy stress $\{\boldsymbol{\sigma}\}$ in the damaged continuum and effective stress $\{\bar{\boldsymbol{\sigma}}\}$ in the equivalent undamaged continuum can be expressed as [12]:

$$\{\bar{\boldsymbol{\sigma}}\} = [\mathbf{M}]\{\boldsymbol{\sigma}\} \tag{1}$$

where $[\mathbf{M}]$ is the effective damage tensor. The non-zero elements of the effective damage tensor [42] are given by

$$\begin{aligned} M_{11} &= 1/(1-D_1), \quad M_{22} = 1/(1-D_2), \quad M_{33} \\ &= 1/(1-D_3), \quad M_{44} = \sqrt{1/(1-D_1)(1-D_2)}, \quad M_{55} \\ &= \sqrt{1/(1-D_1)(1-D_3)}, \quad M_{66} = \sqrt{1/(1-D_2)(1-D_3)} \end{aligned} \tag{2}$$

where D_1, D_2, D_3 are the damage variables along the principal material directions. The constitutive relations for damaged and equivalent undamaged configurations are given as:

$$\{\boldsymbol{\sigma}\} = [\bar{\mathbf{C}}]\{\boldsymbol{\varepsilon}\}, \quad \text{and} \quad \{\bar{\boldsymbol{\sigma}}\} = [\mathbf{C}]\{\bar{\boldsymbol{\varepsilon}}\} \tag{3}$$

where $[\bar{\mathbf{C}}]$ and $[\mathbf{C}]$ are constitutive matrices of damaged and undamaged continua, respectively, $\{\boldsymbol{\varepsilon}\}$ and $\{\bar{\boldsymbol{\varepsilon}}\}$ are respective strains. The principle of equivalence of strain energy in the damaged and undamaged configuration is given by [10]:

$$\frac{1}{2}\{\boldsymbol{\sigma}\}^T[\bar{\mathbf{C}}]^{-1}\{\boldsymbol{\sigma}\} = \frac{1}{2}\{\bar{\boldsymbol{\sigma}}\}^T[\mathbf{C}]^{-1}\{\bar{\boldsymbol{\sigma}}\} \tag{4}$$

Substituting Eq. (1) in Eq. (4), the constitutive matrix of damaged state $[\bar{\mathbf{C}}]$ can be expressed in terms of the principal damage variables and constitutive matrix of undamaged state $[\mathbf{C}]$ as:

$$[\bar{\mathbf{C}}] = [\mathbf{M}]^{-1}[\mathbf{C}][\mathbf{M}]^{-T} \tag{5}$$