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# Elasticity solution of functionally graded circular and annular plates integrated with sensor and actuator layers using differential quadrature

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ABSTRACT

Based on three-dimensional theory of elasticity axisymmetric static analysis of functionally graded circular and annular plates imbedded in piezoelectric layers is investigated using differential quadrature method (DQM). The plate has various edges boundary conditions and its material properties are assumed to vary in an exponential law with the Poisson ratio to be constant. This method can give an analytical solution along the graded direction using the state space method (SSM) and an effective approximate solution along the radial direction using the one-dimensional DQM. The method is validated by comparing numerical results with the results obtained in the literature. Both the direct and the inverse piezoelectric effects are investigated and the influence of piezoelectric layers on the mechanical behavior of plate is studied. The effects of the gradient index, thickness to radius ratio, and edges boundary conditions on the static behavior of FG circular and annular plates are investigated.



## 1. Introduction

Functionally graded material is advanced structural materials, in which the material properties are graded but continuous particularly along the thickness direction to overcome the problem associated with the interfaces in traditional composite materials due to the abrupt change of the materials properties. The research methods for FGM circular plates include simplified theories, approximate theories, numerical method, and exact analysis. Reddy et al. [1] and Wang et al. [2] studied circular/annular plates based on the first-order shear deformation plate theory. Ma and Wang [3] presented relationships between axisymmetric bending and buckling solution of FGM circular plates based on third-order plate theory and classical plate theory. Prakash and Ganapathi [4] presented asymmetric flexural vibration and stability analysis of FGM circular plates in thermal environment by using finite element method. Three-dimensional free and forced vibration analysis of FGM circular plate with various boundary conditions and variation of material properties according to the exponential law was carried out by Nie and Zhong [5]. Semi-analytical solution of axisymmetric bending of two-directional functionally graded circular and annular plates was derived by Nie and Zhong [6] assuming the variation of material properties according to exponential law in both the thickness and radial directions. Ferreira et al. [7] performed static analysis of functionally graded plates by using the collocation method, the radial basis functions and a higher-order shear deformation theory. Bending of transversely isotropic circular plates with elastic compliance coefficients being arbitrary functions of the thickness coordinate, under transverse load was investigated by Li et al. [8]. Three-dimensional free vibration of functionally graded annular plates with different boundary conditions using Chebyshev–Ritz method was investigated by Dong [9]. Free and forced vibration of FGM annular sector plates with simply supported radial edges and variation of material properties according to the simple exponential law was carried out by Nie and Zhong [10] by using state space differential quadrature method. Semi-analytical solution for nonlinear free axisymmetric vibration of a thin circular functionally graded plate in thermal environment based on von-Karman's dynamic equations was presented by Allahverdizadeh et al. [11]. A semi-analytical approach for nonlinear free and forced axisymmetric vibration of a thin circular functionally graded plate was developed by Allahverdizadeh et al. [12] by using assumed-time-mode method and Kantorovich time averaging technique. Malekzadeh et al. [13] discussed in-plane free vibration characteristic of FGM thin-to-moderately thick deep circular arches in thermal environment. Lim et al. [14] discussed temperature-dependent in-plane vibration of functionally graded circular arches based on the two-dimensional theory of elasticity. Free vibration analysis of moderately thick shear deformable annular functionally graded plate coupled with piezoelectric layers with open circuit based on Kirchhof plate model was presented by Ebrahimi et al. [15]. Based on the Levinson plate theory, Sahraee

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[16] investigated thick circular sector plates by using first-order shear deformation theory. Axisymmetric bending and stretching of FG circular plates subjected to transverse loading was analyzed by Sahraee and Saidi [17]. Santos et al. [18] presented axisymmetric free vibration analysis of functionally graded cylindrical shells by using finite element and three-dimensional theory of elasticity. Based on three-dimensional theory, Yun et al. [19] investigated the axisymmetric bending of FG circular plates under arbitrary transverse loads by using the direct displacement method. Axisymmetric bending of transversely isotropic and functionally graded circular plates under arbitrary transverse load using direct displacement method was investigated by Wang et al. [20]. An exact closed-form solution for free vibration of circular and annular moderately thick FG plates based on the Mindlin's first-order shear deformation plate theory was presented by Hosseini-Hashemi et al. [21]. Based on first-order shear deformation theory, Roque et al. [22] studied transient response of FG plates and shells by using meshless numerical method for space domain and Newmark algorithm for the time domain. Geometric nonlinear analysis of functionally graded plates and shells was investigated by using the Marguerre shell element to incorporate the graded properties across the thickness [23]. Free vibration analysis of thin circular and annular plates using Hamiltonian approach was investigated by Zhou et al. [24]. Sburlati and Bardella [25] obtained three-dimensional elasticity solution for FG thick circular plates by using potential function. Golmakani and Kadkhodayan [26] presented axisymmetric nonlinear bending analysis of annular functionally graded plate using third-order shear deformation theory. Nie and Zhong [27] presented frequency analysis of multi-directional FGM annular plates by using state space differential quadrature method based on three dimensional theory of elasticity. Recently, by exploiting the converse and direct piezoelectric effects of piezoelectric materials as distributed actuators or sensors in the FGM circular/annular plate, smart structures has been made with capabilities of self-controlling and self-monitoring. Literature survey shows that considerable research work has been carried out to investigate the bending and vibration behavior of such smart structures. Ebrahimi and Rastgoo [28] investigated analytically free vibration of thin circular functionally graded material (FGM) plates with two uniformly distributed actuator layers made of piezoelectric material based on the classical plate theory (CPT). Ebrahimi and Rastgoo [29] presented analytical solution for free flexural vibration of annular FGM plate integrated with piezoelectric layers. Static analysis of functionally graded, transversely isotropic, magneto-electro-elastic circular plate under uniform mechanical load was discussed by Li et al. [30]. In this investigation they represented displacements and electric potential by appropriate polynomials in the redial coordinate. Vibration analysis of a circular steel substrate surface bonded by a piezoelectric layer was presented by Wu et al. [31]. Vibration analysis of piezoelectric coupled to the thick annular FGM plates subjected to different combinations of soft simply supported, hard simply supported and clamped boundary conditions at the inner and outer edges of the annular plate on the basis of the Reddy's third-order shear deformation theory(TSDT) was presented by Hosseini-Hashemi et al. [32]. Wang et al. [33] derived analytical solution for a three-dimensional transversely isotropic axisymmetric multilayered magneto-electro-elastic circular plate with simply supported boundary conditions. Static analysis of FGM circular/annular plate embedded in piezoelectric layers with arbitrary edges boundary condition based on 3D theory of elasticity has not been presented yet.

In this paper, piezoelastic solution of FGM circular/annular plate under pressure and electrostatic excitation is presented analytically along the graded direction using the state space method (SSM) and a numerical solution along the radial direction using the one-dimensional DQM.

## 2. State space formulation

Consider a FGM circular/annular substrate plate with outer radius $r_i$, inner radius $r_o$, and thickness h, bonded with piezoelectric actuator and sensor on its top and bottom surfaces which is subjected to an axisymmetric transverse mechanical load (uniform pressure) and electric excitation as shown in Fig. 1. A cylindrical coordinate system $r$, $\theta$, $z$ with the origin $o$, on the center of the bottom plane is employed to describe the plate behavior.

### 2.1. Piezoelectric layers

The constitutive equations and electric displacement-strain relations for an orthotropic piezoelectric layer in reference coordinate system $(r,\theta,z)$ are

$$\sigma = C\varepsilon - e^T E \quad (1.a)$$

$$D = e\varepsilon + \eta E \quad (1.b)$$

where $\sigma = \{\sigma_r \quad \sigma_\theta \quad \sigma_z \quad \tau_{rz}\}^T$, $\varepsilon = \{\varepsilon_r \quad \varepsilon_\theta \quad \varepsilon_z \quad \gamma_{rz}\}^T$, $E = \{E_r \quad E_z\}^T$, $D = \{D_r \quad D_z\}^T$,

$$\eta = \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_3 \end{bmatrix}, \quad e = \begin{bmatrix} 0 & 0 & 0 & e_4 \\ e_1 & e_2 & e_3 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 \\ C_{12} & C_{22} & C_{23} & 0 \\ C_{13} & C_{23} & C_{33} & 0 \\ 0 & 0 & 0 & C_{44} \end{bmatrix}$$

$C$, $E$, $D$, $e$ and $\eta$ are material elastic constants, electric field, electric displacement, piezoelectric constants and dielectric constants, respectively.

In the absence of body forces, the equilibrium equations and the charge equation of electroelastics are

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0,$$
$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0 \quad (2)$$

and

$$\frac{\partial D_r}{\partial r} + \frac{D_r}{r} + \frac{\partial D_z}{\partial z} = 0 \quad (3)$$

The linear relations between the strain and the mechanical displacement, the electric field and the electric potential are

$$\varepsilon_r = \frac{\partial u_r}{\partial r} \quad \varepsilon_z = \frac{\partial u_z}{\partial z} \quad \varepsilon_\theta = \frac{u_r}{r} \quad \gamma_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \quad (4)$$

and

$$E_r = -\frac{\partial \varphi}{\partial r} \quad E_z = -\frac{\partial \varphi}{\partial z} \quad (5)$$

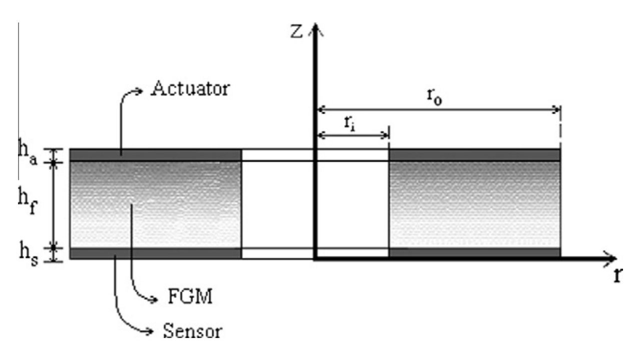


**Fig. 1.** Geometry and coordinates of the laminated plate.