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Influence of geometrical parameters on the elastic response of unidirectional composite materials

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ABSTRACT

A numerical approach to predict the elastic properties of composite materials departing from the properties of the individual constituents is presented. Using a recently proposed algorithm which generates a random distribution of fibres emulating the real distribution in the transverse cross-section of composite materials, an estimate of the elastic properties is obtained by performing volumetric homogenisation of the results from micromechanical analyses. The influence of different geometrical parameters used in the generation of the random distribution of fibres is analysed, namely, the dimensions of the representative volume element, the fibre radius, and the interval between neighbouring fibres.

1. Introduction

The use of composite materials has been flourishing in the last decades. The possibilities awarded by this class of materials are almost endless and top-ranked industries have been profoundly working in developing novel techniques and processes to increase the level of confidence in the application of composite materials to real world problems.

One of the difficulties encountered in the application process of composites is the characterisation of the material behaviour, namely the identification of its elastic properties from the properties of the different constituents. Due to their microscopic heterogeneity, composite materials present themselves as transversely isotropic materials, thus being necessary to identify at least five independent elastic properties to fully characterise the behaviour of the homogenised composite and establish its stiffness tensor.

Throughout the years, an array of analytical approaches has been proposed. Making use of the elastic properties of the constituents of the composite material and the volume fraction of each constituent, different authors proposed closed form equations which provide an estimate of the elastic properties of the composite. Voigt [1] proposed a *rule of mixtures* by assuming that the strains are constant throughout the composite (in both fibre and matrix). A few years later, Reuss [2] proposed what became known as the *inverse rule of mixtures* by assuming that the stress tensors would remain constant in both fibre and matrix. These two conditions establish an upper and a lower bound on the stiffness coefficients of the material [3].

Using the minimum theorems of elasticity, Hashin and Shtrikman [4] developed a set of tighter and more meaningful bounds than those of Voigt and Reuss for isotropic materials with arbitrary internal geometry. Later, Hashin [5] and Hill [6] deduced equations for the bounds of transversely isotropic composites with isotropic constituents.

An alternative model which accounts for the internal geometry of the composite was developed by Hashin and Rosen [7]. The model admits the existence of an assemblage of concentric cylinders, made of two phases: one central part representing the fibre, and one annulus surrounding it representing the matrix.

Mori and Tanaka [8] proposed a method assuming that the average strain in a single reinforcement is related to the average strain in the matrix by a fourth order tensor. This tensor states the relation between the uniform strain in a single reinforcement embedded in an infinite matrix with an imposed uniform strain at infinity.

With the development of computer technology, numerical methods became available for the determination of the elastic properties of advanced composites. In a first approach, Sun and Vaidya [9] considered that the fibres are distributed in a perfectly periodic system. Square and hexagonal arrangements of fibres were used in the study of elastic properties of composites. This pioneering work was improved and extended to the determination of the strength properties of composites [10–13].

However, it is not possible to manufacture composites with such fibre arrangement and it has been seldom demonstrated that a random distribution of fibres always provides closer estimates to experimental data [14]. More importantly, the use of periodic boundary conditions (PBCs) allows for an elimination of edge effects [15] and the results obtained by PBCs are always bounded

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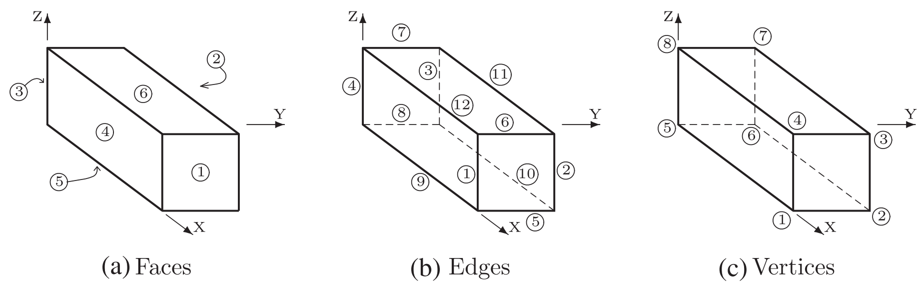

Fig. 1. Numbering on RVE for application of PBCs.

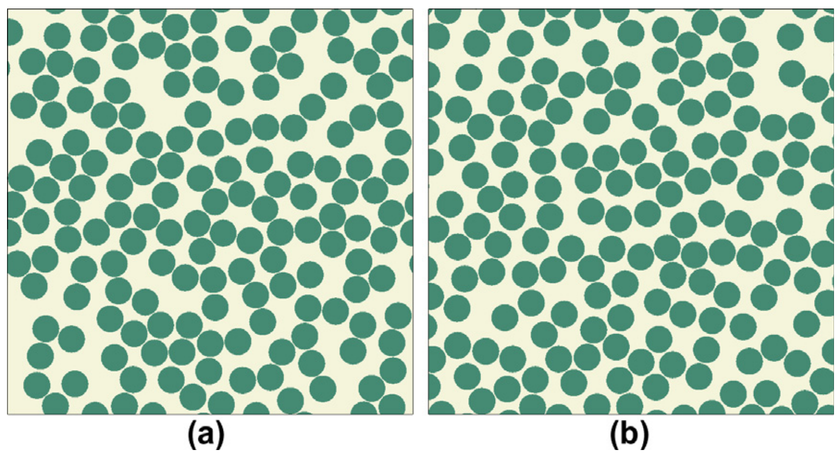

Fig. 2. Example of fibre distributions with (a) and without (b) material periodicity.

Table 1
Elastic properties.

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	G_{23} (GPa)
AS4 carbon fibre	225	15	15	0.2	7
3501-6 Epoxy matrix	4.2	4.2	1.567	0.34	1.567

by those obtained by displacement and traction boundary conditions [16,17]. The analysis of randomly distributed fibres in the transverse cross-section of composites in a representative volume element (RVE) of the real material has been the current trend of work in recent contributions (Canal et al. [18], for example).

Under the lights of such conclusions, the present contribution makes use of a recently proposed algorithm – RAND_uSTRU_GEN [19] – to quickly and efficiently generate a random distribution of fibres. Afterwards, a set of pre-processing techniques create a RVE in which PBCs can be applied and the computation of the elastic constants easily performed. The influence of several geometric parameters required in the implementation of RAND_uSTRU_GEN is studied. Conclusions and recommendations for future analyses are withdrawn from these parametric studies.

2. Periodic boundary conditions

Periodic boundary conditions force such a deformation on the volume element that the displacement of one of the nodes belonging to one edge must be related to the displacement of the corresponding node in the opposite edge.

Barbero [20] provides a set of kinematic constrains that allow for the application of PBCs in a three-dimensional (3D) RVE. The choice for 3D over a more simpler 2D RVE is determined by the desire to study the influence of all degrees of freedom and loading conditions, longitudinal and transverse, as well as the interaction between them. The implementation of PBCs requires that all kinematic constraints must be applied to opposite nodes on the faces, edges and vertices of the RVE. Not only the degrees of freedom of these nodes are variables in these constraints but also the far-field applied strains. Depending on which position the nodes are – faces, edges or vertices – a different set of constraints must be applied to its degrees of freedom. These constraints can be incorporated in a finite element analysis by using linear multi-point constraints [21].

2.1. Faces

Fig. 1a shows the location and numbering used for the faces of the RVE to apply PBCs.

Each node positioned on one face will have its degrees of freedom combined with a node placed on the opposite face. The numbering used for the faces in Eqs. (1) is established according with Fig. 1a [20]:

$$\begin{aligned} u_i^1 - u_i^3 - c\varepsilon_{i1}^0 &= 0 \\ u_i^2 - u_i^4 - a\varepsilon_{i2}^0 &= 0 \\ u_i^6 - u_i^5 - b\varepsilon_{i3}^0 &= 0 \end{aligned} \quad (1)$$

In Eq. (1), u_i^n represents the degree of freedom i of a node in face n. The variables a, b, and c represent the dimension of the RVE in the y, z, and x directions, respectively. The applied far-field strain components are represented by ε_{ij}^0. The far-field strain tensor is a symmetric tensor and is defined in terms of tensorial shear strains.