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Free vibration analysis of multilayered composite cylinder consisting fibers with variable volume fraction

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ABSTRACT

In this paper, free vibration of a fiber reinforced composite cylinder in which volume fraction of its fibers vary longitudinally, is studied using a semi-analytical method. The distribution of volume fraction of fiber in base matrix is based on power law model. A micromechanical model is employed to represent its mechanical properties including elastic and physical properties of this composite cylinder. In addition, kinematically the first order shear deformation shell theory is employed for strain field. Then, weak form formulation and spatial approximations of variables are utilized to discretize the equations of motion. Different problems are solved in which primarily the validity of the results obtained for natural frequencies are evaluated by those similar results reported in the literature and with other commercial F.E. code for different boundary conditions. Furthermore, for different values of volume fractions and under various boundary conditions, computed natural frequencies of this composite cylinder with variable volume fraction of fiber are compared with the traditional one in which the volume fraction of fiber is constant throughout the structure. In spite the fact that average volume fraction of fiber and the layer fiber orientation are the same in both configurations, numerical results show that using variable volume fraction of fiber affects the shell natural frequencies and its mode shapes.

1. Introduction

It is known that shells with cylindrical geometry are one of the most applicable structural elements in different parts of various industries such as construction engineering, aerospace and aeronautical industries, as well as petroleum and petrochemical industries [1]. Thus in-depth knowledge of not only static, but also dynamic behaviors of them is crucial and vital in the design process. One of the most important design parameter in studying dynamic behavior of a structure is to find its natural frequencies. Failure to consider this parameter may end up to catastrophic results because of the resonance phenomena. On the other hand, as the environmental and loading conditions become harsher and harsher the usage of ordinary materials for such geometries could not have almost met all necessary design requirements. As a result, employing new materials with better performance, as substitutions for common low grade materials have received high attention. In this regard for cylindrical shells, fiber reinforced composites as one of new materials with distinct mechanical features, including high specific stiffness and strength [2], have been the main subject of so many researches in the last two decades.

So far, number of different shell theories are employed to study the vibration of fiber reinforced composite cylindrical shells. Most of them are based on Kirchhoff–Love's hypothesis, known as Classical Shell Theories (CST), in which the transverse shear deformations are not taken into account in the body deformations. Studies in this field have been carried out by Darvizeh and Sharma [3], Soldatos [4], Narita and Ohta [5], Lam and Loy [6] and Zhang [7]. Based on Reddy's claim [8], *Kirchhoff–Love's hypothesis is expected to yield adequately accurate results if the radius-to-thickness ratio in the cylindrical shell is large, the dynamic excitations are within the low-frequency range and the material anisotropy is not severe,* otherwise classical shell theories result in an overestimation of natural frequencies. Since these conditions are not always provided, other theories including First-Order Shear Deformations Theory (FSDT) and Higher-Order Shear Deformations Theories (HSDT) are introduced by many researchers, in which the effect of transverse shear deformations is considered in body deformation. Dependency of accuracy of results on shear correction factors in FSDT due to the assumption of constant transverse shear strains in the thickness direction motivated some researchers to use HSDT [14,15]. Nevertheless, the shear correction factors proposed for circular cylinder shells are still common and broadly used by researches and the results from both FSDT and HSDT are quite close and similar.

Studies on vibration of fiber reinforced composite cylindrical shells based on FSDT can be found in works of Nosier and Reddy

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[9], Lam and Qian [10] and Toorani and Lakis [1,2,11], as well as Hufenbach et al. [12] and Jafari et al. [13]. A review on advances made in vibration and dynamic analysis of composite shells can be found in work of Qatu et al. [16]. It should be noted that proposed analytical methods in all above mentioned studies are only applicable to some special cases, in which either boundary conditions or the layer fiber orientation are peculiar. For most complicated cases, usually there is no analytical solution hence, semi-analytical and numerical methods such as finite element that are able to cover all different complicated boundary and fiber layout conditions are widely used by some researchers [8,17].

In recent years, studies and researches on vibration of fiber reinforced composite have been shifted and focused on optimizing the vibration characteristic of such structures by changing the layer fiber orientation [17]. Another method which newly proposed by Oyekoya et al. [18,19] is to use fibers with variable volume fraction in the composite structure. They implemented this idea in a composite plate in which the fiber volume fraction varies according to the power law model and they illustrated that natural frequencies will increase in this plate with clamped-free boundary condition compared to similar plate in which the volume fraction of fiber is constant throughout the plate [18]. However, in their study in order to have a common base for their comparison the densities of matrix and fiber are considered equal.

In this paper, the idea of employing composite with variable volume fraction of fiber following power law changes is utilized in a multilayered cylindrical shell and its impact on natural frequencies of such structure under various boundary conditions is studied. For this purpose, the first order shear deformation shell theory is employed. Then by employing weak form formulation and spatial approximations of variables by Lagrange interpolation functions, the partial governing equations of motion of the shell are discretized into the finite element form. In numerical examples, the results for natural frequencies of traditional composite cylinders under various boundary conditions computed from proposed method are primarily validated by existing results in the literature, as well as ANSYS Software. Then, effects of changing volume fraction of fibers on natural frequencies and mode shapes are studied.

2. Modeling and formulation

Consider a multilayered fiber reinforced composite cylinder in which volume fraction of its fiber varies along its longitudinal direction and each lamina consists of fibers laid in θ direction relative to longitudinal axis, x, of the cylinder, Fig. 1. In this section governing equations of motion for free vibration analysis of this composite cylinder are presented.

2.1. Mechanical properties

According to the proposed micromechanical model by Sinha [20], elastic properties including Young modules (E_1, E_2), Poisson's ratios (υ_{12}, υ_{13}) and shear modules (G_{12}, G_{13}, G_{23}) for an orthotropic lamina reinforced by fiber in the material principal directions, (Fig. 2), can be expressed by Eqs. (1)–(5) as functions of fiber and base matrix mechanical properties, as well as their volume fractions including fiber, V_f and base matrix, V_m [20].

$$E_1 = E_f V_f + E_m V_m + \frac{4(\nu_m - \nu_f)^2 K_f K_m G_m V_m V_f}{(K_f + G_m)K_m + (K_f - K_m)G_m V_f}, \tag{1}$$

$$\nu_{12} = \nu_{13} = \nu_f V_f + \nu_m V_m + \frac{(\nu_m - \nu_f)(K_m - K_f)G_m V_m V_f}{(K_f - G_m)K_m + (K_f - K_m)G_m V_f}, \tag{2}$$

$$G_{12} = G_{13} = G_m \frac{[(G_f + G_m) + (G_f - G_m)V_f]}{[(G_f + G_m) - (G_f - G_m)V_f]}, \tag{3}$$

$$G_{23} = \frac{G_m[K_m(G_f + G_m) + 2G_f G_m + K_m(G_f - G_m)V_f]}{[K_m(G_f + G_m) + 2G_f G_m - (K_m + 2G_m)(G_f - G_m)V_f]}, \tag{4}$$

$$E_2 = \frac{1}{\frac{1}{4K} + \frac{1}{4G_{23}} + \frac{\nu_{12}^2}{E_1}}, \tag{5}$$

where E_f, E_m, υ_f, υ_m, G_f, G_m, K_f and K_m are Young modules, Poisson ratio, shear and bulk modules of fiber and base matrix, respectively and K is the plane module of the lamina and can be computed from following equation:

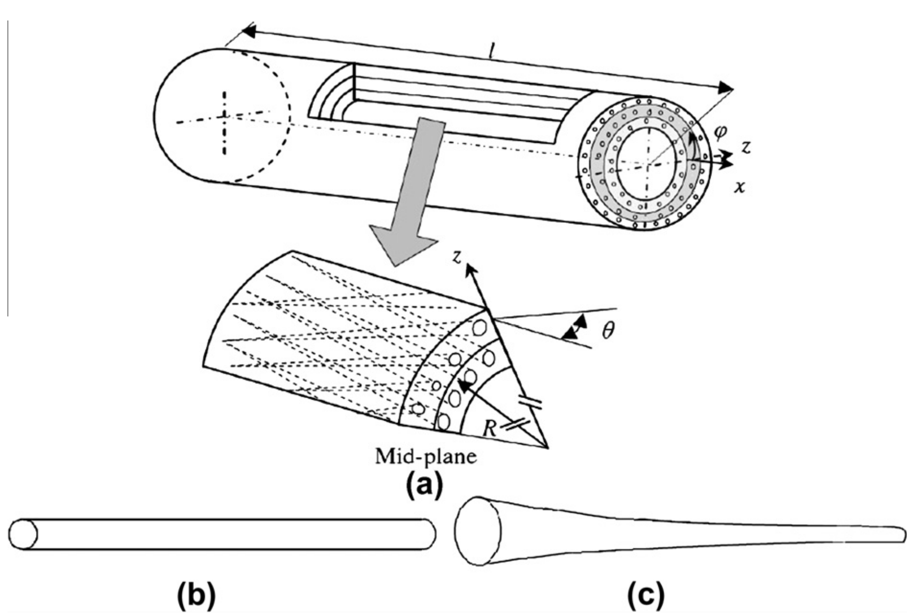

Fig. 1. (a) Schematic view of a composite cylinder cosisting fiber with variable volume fraction. (b) A fiber with constant cross-section in traditional composite. (c) A fiber with variable cross-section in composite with variable volume fraction of fiber.