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Variational asymptotic modeling of the thermomechanical behavior of composite cylindrical shells ☆

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ABSTRACT

An efficient shell model is developed for analyzing one-way coupled thermomechanical behavior of composite cylindrical shells by using the variational asymptotic method (VAM). Taking advantage of the smallness parameter inherent in the shell structure, the VAM is applied to rigorously decouple the 3-D, thermoelasticity problem into a 1-D through-the-thickness analysis and a 2-D shell analysis. The through-the-thickness analysis servers as a link between the original 3-D analysis and the shell analysis by providing a constitutive model for the shell analysis and recovering the 3-D field variables in terms of global responses calculated by the shell analysis. The present model is valid for large displacements and global rotations and can capture all the geometric nonlinearity of a shell when the strains are small. A few examples are used to validate this model.

1. Introduction

Composite materials are widely used in engineering practices due to their excellent strength to weight and stiffness to weight ratio, flexibility of tailoring their properties through engineering the microstructure and improving manufacturing technology. However, the heterogeneity and anisotropy of such materials make the traditional analysis method used for designing homogeneous and isotropic structures obsolete. Moreover, structures made with composite materials are more sensitive and vulnerable to temperature change than their isotropic counterparts. The reason is that the thermal expansion coefficients of different constituents of the material are usually dramatically different from each other resulting in high stresses due to temperature changes from stress free environment.

Although all structures made of composite materials can be described using 3-D continuum mechanics, exact solutions exist only for a few specific problems with very idealized material types, geometry and boundary conditions [1]. For more realistic cases, 3-D numerical simulation tools such as ANSYS and ABAQUS are often used to find approximate solutions. However, this approach is computation intensive and usually used in the detailed analysis due to its prohibitive computational cost.

To use composite structure effectively, an efficient yet accurate model for shells made of such materials needs to be developed. In view of the fact that the thickness is much smaller than the other two, analysis of such structures can be simplified using 2-D models. Although many 2-D models have been developed to analyze laminated shells treating different topics, most of them rely on some apriori kinematic assumptions.

An example may be found in the application of Classical Lamination Theory (CLT) in studying the dynamic stability of laminated cylindrical shells under combined static and periodic axial forces [2]. CLT ignores transverse shear effects and provides reasonable results only for very thin shells. Moreover, in CLT, both plane strain and plane stress are assumed which we know will not be true at the same time for materials having nonzero Poisson's ratios.

A number of shear deformation theories have been developed to overcome some drawbacks of CLT, with the simplest of which being the First-Order Shear-Deformation Theory [3] (FOSDT, equivalent to Reissner–Mindlin theory for shells made of isotropic homogeneous materials), where a constant distribution of shear strain through the thickness is assumed and a shear correction factor is required to account for the deviation of the real shear strain from the assumed constant one. The dependence of the shear correction factor on the geometry and material of the shell makes it difficult to guarantee the accuracy of FOSDT.

By expanding the displacement field of the shell using higher-order polynomials, the High-Order Shell Theory (HOST) which can account for both transverse normal and shear deformations without relying on shear correction factors is developed [4]. Zhen

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Nomenclature

α	column matrix of the 3-D thermal expansion coefficients
$\delta\overline{W},\ \delta\overline{W}_{2D}$	3-D and 2-D virtual work, respectively
Γ_{ij}	3-D strain tensor
γ	column matrix of transverse strain measures
μ	characteristic magnitude of the elastic constants
Ω	the reference surface of shell
ϕ	geometric correction for shell structure
ρ	mass per unit area
$\mathbf{R}$	strain measures of Reissner–Mindlin shell
$\mathbf{r}$	position vector from O to the point located by x_α on the reference surface
$\varepsilon_{\alpha\beta}$, $\kappa_{\alpha\beta}$	2-D generalized shell strains
A_α	*Lamé* parameters
C_{ij}	matrix of direction cosines
D	3-D material matrix
f_i, m_α	generalized forces and moments, respectively
h, R, φ	thickness, radius and included angle of the shell, respectively
K, K_{2D}	3-D and 2-D kinetic energy, respectively
$k_{\alpha\beta}$	out-of-plane curvatures
l	characteristic wavelength of the shell deformation
n	a small parameter used to denote the order of strains
p_i, τ_i, β_i	applied body force and tractions applied on the top and bottom surfaces, respectively
T	temperature difference from the stress-free temperature
U_i	the displacements of reference surface in Cartesian coordinate x_i
U, U_{2D}	3-D and 2-D Helmholtz free energy density, respectively
w_i	warping functions for the normal-line element
x_i	Cartesian coordinate

and Wanji [5] have recently developed a higher-order model to accurately predict the displacements and stresses in laminated shells, in addition the thermal bending is also considered. Khdeir et al. [6,7] compared the HOST12 (12 parameters, cubic expansion in the z direction for each displacement component) with the HSDT (Higher Shear Deformation Theory) and solved exactly the thermoelastic governing equations for laminated shells. Brischetto and Carrera [8] extended the Carreras Unified Formulation (CUF) and the Principle of Virtual Displacements (PVDs) to derive differential governing equations for the thermal analysis of shells with constant radii of curvature. However, as indicated by Bian et al. [9] and Das et al. [10] for laminated shells, models based on HOST cannot capture the discontinuous slope of in-plane and transverse displacement components in the thickness direction. To overcome the drawbacks of HOST, Layerwise Theories (LWTs) [11] have been developed to produce reasonable results at the cost of complex models and expensive computation. Displacements interlaminar continuity can be imposed more conveniently by employing interface values as unknown variables.

Despite of the popularity of the aforementioned methods in analyzing laminated shell, those approaches have two major disadvantages: (1) the apriori assumptions which are naturally extended from the analysis of isotropic homogeneous structures cannot be easily justified for heterogeneous and anisotropic structures; (2) it is difficult for an analyst to determine the accuracy of the result and which assumption should be chosen for efficient yet accurate analysis for a particular shell.

In this paper, an efficient high-fidelity shell model for composite cylindrical shells is constructed by the variational asymptotic method (VAM). The 3-D elasticity problem is rigorously divided into two problems: a nonlinear, 2-D, shell analysis over the reference surface to obtain the global deformation and a linear, 1-D, through-the-thickness analysis to provide the 2-D generalized constitutive law and the recovering relations to approximate the original 3-D results. The non-uniqueness of the asymptotic theory correct up to a certain order is used to cast the asymptotically correct second-order energy into a Reissner–Mindlin type model, in order to account for transverse shear deformation. The present theory extends a simple and accurate model developed recently for composite laminates by Yu et al. [12,13] so that the composite cylindrical shell can be treated in the same framework. Since the procedure is quite similar, the authors have chosen to repeat some formulae and texts from their previous publications to make the present paper more self-contained.

2. Asymptotically correct geometrically nonlinear shell theory

2.1. 3-D formulation

The elastodynamic behavior of a solid is governed by the extended Hamilton principle:

$$\int_{t_1}^{t_2}\int_{v}[\delta(K-U)+\delta\overline{W}]\mathrm{d}v\mathrm{d}t=0 \tag{1}$$

where δ is the variation symbol, v denotes the volume of the undeformed body, t_α are arbitrary fixed times (here and throughout the paper, Greek indices assume values 1 and 2 while Latin indices assume 1, 2, and 3. Repeated indices are summed over their range except where explicitly indicated), K and U are the kinetic and Helmholtz free-energy density, respectively, and $\delta\overline{W}$ is the virtual work of applied loads. The overbar is used to indicate that the virtual work does not necessarily represent variation of functionals.

Since only the one-way thermomechanical coupling is considered in present work and the temperature change due to the deformation of the plate is negligible, the Helmholtz free-energy functional [14] without quadratic terms of temperature can be used to carry out the analysis. This leads:

$$U=\left(\frac{1}{2}\Gamma^{\mathrm{T}}D\Gamma-\Gamma^{\mathrm{T}}D\alpha T\right)\phi \tag{2}$$

where D is the 6×6 material matrix condensed from the 3-D fourth-order elasticity tensor expressed in the global coordinate system x_i, α is the 6×1 column matrix of the 3-D thermal expansion coefficients, T is the temperature difference from the stress-free temperature, Γ denotes the 3-D strain field, and ϕ is defined as geometric correction for shell structures,

$$\phi=1+x_3(k_{11}+k_{22})+O(h^2/l^2) \tag{3}$$

with h, l are thickness and characteristic wavelength of the shell deformation, respectively, $k_{\alpha\beta}$ refers to the usual out-of-plane curvatures and $k_{12}=k_{21}=0$ because the coordinates are chosen to the lines of curvatures (see Fig. 1).

After the dimensional reduction, the Hamilton's extended principle can be reformulated for the reference surface of shell as:

$$\int_{t_1}^{t_2}\int_{\Omega}\left[\delta(K_{2D}-U_{2D})+\delta\overline{W}_{2D}\right]\mathrm{d}\Omega\mathrm{d}t=0 \tag{4}$$