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Selection of appropriate multilayered plate theories by using a genetic like algorithm

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ABSTRACT

This paper evaluates a number of classical and refined two-dimensional theories for the analysis of metallic and composite layered plates. Thin-plate, shear deformation and higher order plate theories are compared for various plate problems related to different mechanical and geometrical boundary conditions (BCs), as well as geometries and staking sequence lay-out. The theories are implemented by referring to a Unified Formulation (UF) proposed by the first author. The UF allows displacement fields with any order *N* in the thickness plate direction to be introduced and any variables in the *N*-order displacement field to be discarded. The finite element method is applied to include anisotropy and complex BCs. The accuracy of given theories for each fixed problem is established in terms of displacement and stress fields. The best plate theories, that is the most accurate plate theories with few computational efforts, is then determined by exploring various possibilities and by selecting appropriate unknown variables upon application of genetic algorithms. A best plate curve is established which shows the best plate theories (number of terms and their meanings) in terms of accuracy. It is concluded that a best plate theory changes with changing geometry, lay-out and BCs. The genetic algorithm used allows the least expensive computational model of each given problem to be detected.

1. Introduction

Metallic and laminated composite panels are used to build much of the structures of modern aerospace vehicles. The rational design and analysis of these panels is a fundamental task for structural analysts. Accurate evaluation of their static, dynamic, buckling, aeroelastic and fatigue response require adequate structural models for both metallic and composite plates. Since the works by Lekhnitskii [1,2], a number of relevant contributions have been made to improve the classical plate theories originally proposed for one-layered isotropic structures. For a complete overview of existing Layer-Wise and Equivalent Single Layer theories see the excellent reviews by Ambartsumian [3], Librescu and Reddy [4], Grigolyuk and Kulikov [5], Kapania and Raciti [6,7], Kapania [8], Noor [9–11], Reddy and Robbins [12], Carrera [13,14], Qatu [15,16], and the books by Librescu [17], Reddy [18] and Qatu [19]. Theories can be classified according to the expansion adopted for unknown variables:

- Classical models are based on Kirchhoff–Love's assumptions: thickness strain as well as transverse shear deformations are neglected. The Classical Lamination Theory (CLT) is pertinent to this group.
- Refined theories are obtained if at least one of Kirchhoff's hypotheses is removed. For example, the Reissner–Mindlin theories, also known as First Order Shear Deformation Theory (FSDT), accounts for a constant through-the-thickness transverse shear deformation. Higher order theories, such as Vlasov's or Hildebrand–Reissner–Thomas's, are based on higher order expansions of the displacement components on the reference surface.

In the case of layered structures the so-called Zig-Zag (ZZ) theories are particularly noteworthy since they include the ZZ effect through-the-thickness variation and the Interlaminar Continuity (IC) of transverse shear and normal stresses within the equivalent single layer approach. A historical overview [20] has established that the ZZ theories can be grouped as: Lekhnitskii's Multilayered Theories (LMTs); Ambartsumian's Multilayered Theories (AMTs) and Reissner's Multilayered Theories (RMTs).

Refined theories increase the number of unknown variables. Such an increase could become prohibitive in the case of the application of computational methods such as the Finite Element Method. Recent works [21–24] have contributed to giving an answer to the following question: for a given problem (geometry, loading, boundary conditions, lamination lay-out) what is the most accurate plate theory in terms of a fixed accuracy? The above mentioned works make use of the Unified Formulation by the first author of this paper [25] to generate governing equations and

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finite element matrices in terms of *fundamental nuclei* whose form does not change with a changing plate theory. In [21], attention is focused on the closed form solution for isotropic and laminated plates, whereas the refined beam is addressed in [22]. Finite element plate cases for isotropic and laminated structures are considered in [23,24], respectively. A so-called 'best plate theory diagram' BPTD was traced for many problems, that is, plate theories were recognized in terms of error on a given parameter (stress, displacement etc.). By tracing the BPTD, the authors noticed that in many cases (especially in the case of laminated plates) the number of 'quasi-optimal' solutions (plate theories close to BPTD) can be very high and these solutions can vastly differ from each other. This fact introduces difficulties in clearly establishing the BPTD. To overcome this problem the present work introduces a genetic type algorithm to build a robust BPTD. Improved plate theories derived from a starting group of theories; is the initial population. Each theory can be considered as an individual with its own DNA and able to determine the solution with a certain degree of error and its own computational cost. Following the rules of biological evolution, described by Darwin in "The Origin of Species" the initial population can evolve and change, generating new individuals which can determine the solution with a lower degree of error and a lower computational cost than their parents. By imposing an evolutionary pressure through "natural selection" it is possible to reward the best theories by identifying a population which defines the Best Theory plate Curve after a certain number of generations. The concept of Genetic Algorithm was developed by Holland and his co-workers in the 1960s and 1970s [26]. An overview of the multiple-objective optimization method using genetic algorithms is presented by Abdullah et al. [27] and Fonseca and Fleming [28]. An overview of the use of genetic algorithms in engineering is presented by Gbor and Anik [29]. The present work is organized as follows: a brief description of the adopted CUF formulations is given in Sections 2–4; the method used to determine the Best Theories Plate Curve is introduced in Section 5; numerical results are provided in Section 6 and the main conclusions are outlined in Section 7.

2. Preliminaries

The coordinate reference frame is shown in Fig. 1, where x and y are the in-plane coordinates while z is the thickness coordinate. The displacement vector, $\mathbf{u}^k$, of a single layer is defined as:

$$\mathbf{u}^k(x,y,z) = \left\{ u_x^k \quad u_y^k \quad u_z^k \right\}^T \tag{1}$$

The superscript "T" represents the transpose operator. Stress and strain components are grouped as follows:

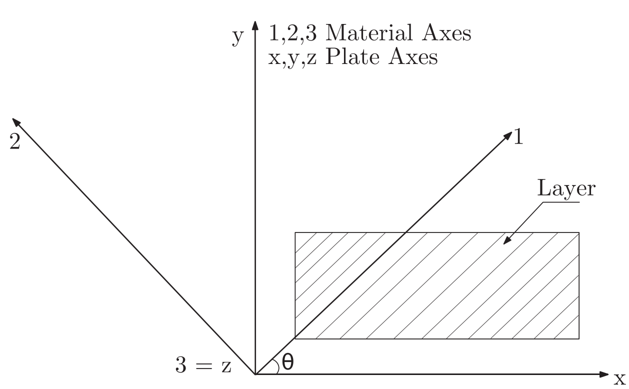

Fig. 1. Coordinate reference system.

$$\begin{aligned}
\boldsymbol{\sigma}_{\boldsymbol{p}}^k &= \left\{ \sigma_{xx}^k \quad \sigma_{yy}^k \quad \sigma_{xz}^k \right\}^T & \boldsymbol{\epsilon}_{\boldsymbol{p}}^k &= \left\{ \epsilon_{xx}^k \quad \epsilon_{yy}^k \quad \epsilon_{xz}^k \right\}^T \\
\boldsymbol{\sigma}_{\boldsymbol{n}}^k &= \left\{ \sigma_{xz}^k \quad \sigma_{yz}^k \quad \sigma_{zz}^k \right\}^T & \boldsymbol{\epsilon}_{\boldsymbol{n}}^k &= \left\{ \epsilon_{xz}^k \quad \epsilon_{yz}^k \quad \epsilon_{zz}^k \right\}^T
\end{aligned} \tag{2}$$

where p indicates the in-plane components and n the out-of-plane components. Linear strain–displacement relations are defined as:

$$\begin{aligned}
\boldsymbol{\epsilon}_{\boldsymbol{p}}^k &= \boldsymbol{D}_p \boldsymbol{u}^k \\
\boldsymbol{\epsilon}_{\boldsymbol{n}}^k &= \boldsymbol{D}_n \boldsymbol{u}^k = (\boldsymbol{D}_{n\Omega} + \boldsymbol{D}_{nz})\boldsymbol{u}^k
\end{aligned} \tag{3}$$

where

$$\boldsymbol{D}_p = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \quad \boldsymbol{D}_{n\Omega} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial y} \\ 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{D}_{nz} = \begin{bmatrix} \frac{\partial}{\partial z} & 0 & 0 \\ 0 & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \end{bmatrix} \tag{4}$$

The stress components in the material reference coordinates are obtained by the constitutive law:

$$\boldsymbol{\sigma}_{\boldsymbol{m}}^k = \boldsymbol{C}\boldsymbol{\epsilon}_{\boldsymbol{m}}^k \tag{5}$$

where C, $\boldsymbol{\sigma_m}$ and $\boldsymbol{\epsilon_m}$ are written in the material coordinates whose explicit expression are not reported here for sake of brevity. They can be found in [24].

3. Plate theory based on Carrera Unified Formulation

In the framework of the Carrera Unified Formulation, the displacement components u_x^k, u_y^k and u_z^k of the k-layers (the total number of layers is indicated by N_l) are postulated, in the z-direction, according to the expansion

$$\begin{aligned}
u_x^k &= F_t u_{xt}^k + F_r u_{xr}^k + F_b u_{xb}^k \\
u_y^k &= F_t u_{yt}^k + F_r u_{yr}^k + F_b u_{yb}^k \quad r = 2,3,\ldots,N \\
u_z^k &= F_t u_{zt}^k + F_r u_{zr}^k + F_b u_{zb}^k
\end{aligned} \tag{6}$$

The subscript t and b denote values related to the top and bottom layer surfaces, respectively. F_t, F_r, and F_b are base functions of z. Through the model (6) the continuity of the displacement can be imposed to the layer interfaces. Eq. (6) can be written in a compact manner as

$$\boldsymbol{u}^k = F_t \boldsymbol{u}_t^k + F_r \boldsymbol{u}_r^k + F_b \boldsymbol{u}_b^k = F_\tau \boldsymbol{u}_\tau^k \qquad \tau = t,r,b; \quad r = 2,3,N \tag{7}$$

where the components of $\boldsymbol{u}_\tau^k$ are

$$\boldsymbol{u}_t^k = \begin{Bmatrix} u_{xt}^k \\ u_{yt}^k \\ u_{zt}^k \end{Bmatrix} \quad \boldsymbol{u}_r^k = \begin{Bmatrix} u_{xr}^k \\ u_{yr}^k \\ u_{zr}^k \end{Bmatrix} \quad \boldsymbol{u}_b^k = \begin{Bmatrix} u_{xb}^k \\ u_{yb}^k \\ u_{zb}^k \end{Bmatrix} \tag{8}$$

From (6) any-order displacement fields, Layer Wise (LW) and Equivalent Single Layer (ESL), can be adopted. Imposing the conditions

$$\begin{aligned}
u_{xt}^k &= u_{x0} \quad u_{xr}^k = u_{xr} \quad u_{xb}^k = u_{x_N} \quad F_t = 1 \\
u_{yt}^k &= u_{y0} \quad u_{yr}^k = u_{yr} \quad u_{yb}^k = u_{y_N} \quad F_r = z^r \\
u_{zt}^k &= u_{z0} \quad u_{zr}^k = u_{zr} \quad u_{zb}^k = u_{z_N} \quad F_b = z^N
\end{aligned} \tag{9}$$

and discarding k it is possible to obtain an ESL displacement field. For example if $N = 4$ it is possible to obtain the following displacement field

$$\begin{aligned}
u_x &= u_{x0} + zu_{x1} + z^2u_{x2} + z^3u_{x3} + z^4u_{x4} \\
u_y &= u_{y0} + zu_{y1} + z^2u_{y2} + z^3u_{y3} + z^4u_{y4} \\
u_z &= u_{z0} + zu_{z1} + z^2u_{z2} + z^3u_{z3} + z^4u_{z4}
\end{aligned} \tag{10}$$

Classical plate theories can also be obtained. The Reissner–Mindlin plate model approximation [30,31], also known as First Order Shear Deformation Theory, FSDT, in the case of laminates, requires two conditions: (1) first-order approximation kinematic fields, (2) the